Exercises on some game theoretical notions
Correction

1 Normal and extended form of games

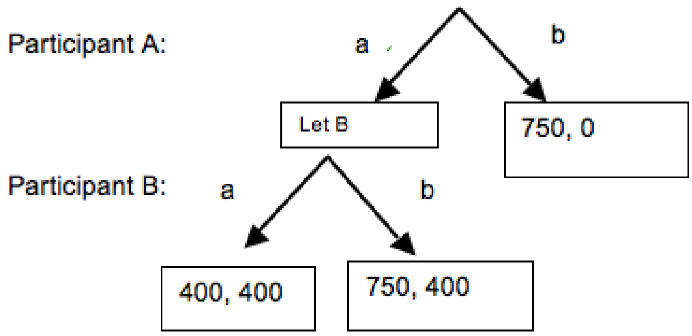
The main idea was to ascribe arbitrary numbers to outcomes, in such a way that their relations (bigger than, smaller than) reflect the constraints spelled out in the text of the exercise. You had then to arrange these numbers in the normal form used in game theory.

Bro. V

Bro. A		Lego	Train
	Lego	1,0	0,1
	Train	0,1	1,0

	Passive	Aggressive
Passive	2, 2	0, 4
Aggressive	4,0	1,1

Remember that an extended form of a game is a representation that looks like an inverted tree of decisions, with the payoffs as the end 'leaves' of the tree. The normal form is a table of payoffs (for two players), with the decisions at the top of the columns and the left of the rows.



One difficulty: to transform a dynamic game represented in an extended form in a normal form game. You had to spell out every possible strategy of the first player as conditional on what the first player would do. This meant that you would have had, for B, to include:

S1: If A chooses a then choose b, else choose b

S2: If A chooses a then choose b, else choose a

S3: If A chooses a then choose a, else choose b

S4: If A chooses a then choose a, else choose a

I know it's a bit tedious: Note that if A has had had three choices, then B strategies would have the form: if A chooses a then ..., and if A chooses b then ... , and if A chooses c then ... He consequently would have had 3^2 possible strategies.

Every cell in the payoff matrix should be filled in.

This gives:

	S1	S2	S3	S4
a	750, 400	400, 400	750, 400	400, 400
b	750, 0	750, 0	750, 0	750, 0

2 Finding pure strategy Nash equilibria

Remember that the methods for finding pure Nash equilibria are either 1. or 2. explained below:

1. Check each set of action profile/strategies: does it satisfy the conditions for being a Nash equilibrium? The conditions are that no player can do better by *unilaterally* changing his decision (action/strategy)

2. (a) Use a normal form of a game.
   (b) Select a player; fix the strategies of other players; mark the best strategies for your selected player responding to the fixed strategies of others. (You can underline the player's payoff).
   (c) Repeat the above in order to find best answers to all the other possible strategies
   (d) Repeat this procedure (a) and (b) for all players.
   (e) Equilibria exist for those cells where all the players' payoff are marked (underlined).

You should then select the cells with the following payoffs:

2.1 Equilibria: 2,2 and 1,1

2.2 Equilibrium: 1,1

2.3 Equilibria: 1,3 and 3,1 and 0,0 (left-bottom corner).

2.4 Equilibrium: 1,1

2.5 No pure strategy Nash equilibrium

2.6 Equilibria: 1,2 and 2,1

In the last exercise (2.7), I wanted to point out that in some cases where everybody cares mostly about the comfort of the other, people can end up being less comfortable than when people selfishly look for their own comfort: with altruistism and politeness, the game was a prisoner's dilemma where both player ended up standing and thus not enjoying the comfort of sitting, but also not enjoying the altuistic pleasure of seeing the other one sited. I thought this paradoxical situation is often encountered in our day-to-day life (e.g. going first through a door).

2.1 Risk and expected utility

In this exercise, you had to compare the expected utility of Gyuri's available choices:

1. Offer €1, keep €9
2. Offer €3, keep €7
3. Offer €7, keep €3

The difficulty of this exercise comes from the fact that this expected utility depends on whether the offer is accepted or refused, which itself depends on the utility that Gyuri's partner can derive from accepting or refusing the offer, which in turn depends on Gyuri's partner's social preferences. You consequently had to go through the following steps:

1. Check what offers would be accepted by inequity averse villagers.
   (a) Calculate with the utility function of inequity averse people specified in the text, what is the utility of accepting each possible offer.
   (b) Compare it with the utility of rejecting the offer, which is zero.
   (c) Conclude whether the offer is accepted or rejected for those who have other regarding preference: it is accepted only if the utility of accepting is higher than zero—the utility of rejecting.
   (d) Do that for every potential offer.
2. For those who have only selfish preference, you already know that they will always accept the offer.
3. Calculate the probability that each offer Gyuri can make is accepted or rejected in view of the frequency of inequity averse and selfish maximizer in the population (.4 and .6 respectively).
4. Calculate the expected utility for each possible offer: it is equal to Gyuri's payoff when the offer is accepted (remember that Gyuri has no social preferences) multiplied by the probability that it is indeed accepted.
5. Compare these expected utility and select the choice that has the highest expected utility.

So ...
Inequity averse utility for accepting the offers are:
<u>Option a</u>

$$u(9,1) = [1-(-0.35)] \times 1 + (-0.35) \times 9 = 1.35 - 3.15 = -1.8$$

<u>Option b</u>

$$u(7,3) = [1-(-0.35)] \times 3 + (-0.35) \times 7 = 4.05 - 2.45 = 1.6$$

<u>Option c</u>

$$u(3,7) = (1-0.2) \times 7 + 0.2 \times 3 = 5.06 + 0.6 = 5.12$$

Inequity averse individuals will turn down offer a), because the utility of it is less than 0, but will accept the other two offers, because their utility is more than 0.

If Gyuri chooses a), then there is 60% possibility that his offer is accepted, and 40% that it won't be, so his expected payoff is

$$0.6 \times 9 + 0.4 \times 0 = 5.4$$

If Gyuri chooses b) or c), then the offer will be accepted, so his (expected) payoffs are 7 and 3, respectively.

Gyuri will consequently choose option b)

3 Mixed Nash Equilibrium

Assume that column's playing L with prob. p

Assume that row's playing U with prob. q

Then the expected payoff for row is:

$$U_{row}(U) = 5p + 0(1 - p)$$
$$U_{row}(D) = 4p + 2(1 - p)$$

Because playing L with probability p is a mixed strategy that belongs to a Nash equilibrium we know that row needs to be indifferent between playing U and D: if not, then row will choose the pure strategy that maximizes his expected payoff, and column can then find a pure strategy that is a best response to row's strategy. But this is not what we are looking for. We're looking for a p such that $0 < p < 1$.

So to find p, one need to solve the equation

$$\begin{aligned} U_{row}(U) &= U_{row}(D) \\ \Rightarrow 5p + 0(1 - p) &= 4p + 2(1 - p) \\ \Rightarrow p &= 2/3 \end{aligned}$$

Same reasoning for q would give $q = 1/3$