

The Death of the Object: *Fin de siècle* Philosophy of Physics

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Science and objectivity are often taken to be coextensive, not least by the scientists themselves. Historians, sociologists, and many philosophers of science tend now to regard this viewpoint as naive, but there is a refined sense in which it is at least defensible. For science might almost be defined as the control of subjectivity. We rarely speak of science unless we find wide agreement about the knowledge, methods, and problems of any given discipline. Overcoming or transcending what is merely personal is presupposed in the making of shared public knowledge. The ability of the sciences to do this, hence to speak often with one voice, is central to their prestige and authority. The suppression of subjectivity has certain unfavorable overtones, which humanistic critics have not failed to point out, but it is almost universally regarded within science, and by the larger public, as a triumph, not a limitation. Objectivity in science means laws and facts. It implies the possibility of real knowledge, shared by a community and valid for everyone. In democratic societies it is greatly valued for its connotations of fairness and impartiality.¹

Given the rhetorical power of the identification of science with objectivity, the disinclination of many late-nineteenth-century physicists to believe they could gain access to independently existing objects appears somewhat puzzling. One naturally interprets it as at least a defensive move, if not an indication of despair. Descriptionism, to use John Heilbron's term,² has surfaced repeatedly in physics since the Middle Ages. In the time of Copernicus the business of mathematical astronomy

did not go beyond predicting and describing, while causes and explanations were left to the higher disciplines of (Aristotelian) physics and theology. Galileo got in trouble by disregarding Bellarmine's injunction to confine himself to description. Newton announced his prohibition against hypotheses to vindicate an inverse-square law of attraction for which he could find no satisfactory mechanism. In each case descriptionism was decidedly a defensive stance. One important role of descriptionism was to keep physics in its place and leave causal explanations to higher authorities. Another was to provide a rationale under which science could be pursued even when there were no answers for the most important questions. This latter was especially common in the late eighteenth and early nineteenth centuries. If natural philosophers could not reach consensus about the causes of electricity, heat, magnetism, and chemical properties, they could at least agree on conventions for measuring and classifying the phenomena.³

The positivistic stance of physics in the late nineteenth century has generally been explained in similar terms. Thomas Kuhn's *Structure of Scientific Revolutions* argued for a crisis of physics preceding the twin revolutions of relativity and the quantum. It has been widely supposed that the world of classical physics was breaking down, that the old explanations no longer worked and new ones had yet to be born. It is, I think, more plausible to point to an efflorescence of competing conceptions of the proper foundation of physics than to a despair about finding foundations at all.⁴ But this was scarcely unprecedented and, in any event, did not amount to a crisis. Indeed, remarkable complacency was possible. Albert Michelson, whose negative experimental results on the ether seemed in retrospect so portentous, is otherwise best known for his pronouncement that henceforth progress in physics should be above all a matter of precise measurement and would occur in the sixth decimal place. Heilbron remarks that in working science, if not in abstract discussion, physicists continued to talk of ethers and atoms with no more uncertainty than of workbenches and colleagues.⁵ Finding, as he does, no pervasive malaise over theory, Heilbron seeks to explain the fashion of descriptionism by examining the relation of physicists to higher authorities. Threatened by charges that they aimed to subvert religion, their espousal of descriptionism was an irenic move, a concession to older values and higher powers. As their need for resources expanded, it was all the more important to avoid the disfavor of the "mandarins of the old culture." This they hoped to accomplish by recognizing and accepting their modest station, describing phenomena rather than barbing reality.⁶ If not despair, then humility tending to sycophancy may be taken to account for the physicists' turn from realism.

Without denying that descriptionism could serve a useful defensive role in dangerous times, I want here to explain how the death of the object provided the rationale for a more optimistic, almost exuberant, view of physics. Even in its defensive meaning, descriptionism aimed to make physics almost impregnable, to confer on it something like the degree of certainty normally associated with mathematics. Far from supporting a rearguard action to exalt subjectivity, this retreat from belief in objects tended to remove sources of controversy and thus to enhance objectivity in the sense of intersubjectivity. More than that, though, the release of physics from all particular objects helped to dissolve the boundaries that confined physics to one aspect of the natural world. What was given up in depth was repaid tenfold in breadth. Thus, far from narrowing the domain of physics, descriptionism made boundaries seem artificial. The two philosophers of science with whom I am mainly concerned here, Ernst Mach and Karl Pearson, aimed explicitly to define the aims and methods of science in a way that could apply equally well to the realm of the human as to that of inorganic nature. They were true predecessors of the Vienna Circle, advocates of unified science under the auspices of positivism.

MODELS, DESCRIPTIONS, AND THE LOCUS OF CERTAINTY

The tenth edition of the *Encyclopaedia Britannica*, appearing in 1902, carried a new entry, "Model," by the Austrian physicist Ludwig Boltzmann. Mach spoke disparagingly of models, and yet the very idea represented a large concession to his way of thinking. For talk of models tended to displace truth claims. "Model," explained Boltzmann, is "a tangible representation . . . of an object." Sculptors and engineers make models to provide a pattern or mold to guide their work. The mathematical and physical sciences, it was now realized, also rely on models, for "our thoughts stand to things in the same relation as models to the objects they represent."⁷ Boltzmann identified the British physicist James Clerk Maxwell as having first properly comprehended the role of models in science. Maxwell, he explained, understood that the true nature of atoms and molecules is "absolutely unknown," and he posited mechanical agents not as true representations, or even hypotheses, but "merely as means by which phenomena could be reproduced, bearing a certain similarity to those actually existing." Models have as much to do with human thinkers as with nature: they "are really a continuation and integration of our process of thought."⁸

Boltzmann here singled out Maxwell's work on molecules, his kinetic theory of gases, as exemplary. This was the main area of Boltzmann's own researches. Maxwell, though, worked out his ideas on models primarily in relation to the theory of electricity. Electrical field theory, arguably the most consequential theoretical shift in nineteenth-century physics, began as nothing more than the working out of models. And Maxwell was quite conscious of this. In the earlier of his two most important papers on electricity, in 1855, he began by explaining the need for "analogies." The multiplication of formulas and phenomena in the science of electricity, he explained, had reached the point where they must somehow be reduced and simplified.

The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis. In this first case we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject. If, on the other hand, we adopt a physical hypothesis, we see the phenomena only through a medium, and are liable to that blindness to facts and rashness in assumption which a partial explanation encourages. . . . In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies, . . . [i.e.] that partial similarity between the laws of one science and those of another which makes each of them illustrate the other.

He tried not to invite suspicion by claiming methodological originality. All the mathematical sciences, he added immediately, are founded on relations of analogy "between physical laws and laws of numbers." Still, it was decidedly unusual to set out, as he did, from a manifest fiction. He compared the transmission of electrical forces to the flow of a fluid through an array of fine tubes. The result was to bypass the usual conception of force as action at a distance and to represent electrical forces in the same terms, and with nearly the same mathematics, as applied to the flow of heat.

Maxwell's argument was unmistakably analogical, but it was not at all frivolous, and he was far from adopting the thoroughly antirealist posture that a wholly fanciful theory is as good as any other if its predictions are borne out. Although Maxwell clearly did not believe that the propagation of electrical forces involved the flow of fluid through tiny pipes, he was convinced that electrical energy was located somehow in an ethereal medium and not merely in capacitors and conducting wires. In this respect he was a good disciple of Michael Faraday, as

indeed the title of his paper, "On Faraday's Lines of Force," suggests. Six years later, in 1861-62, he published another, still more important paper that expressed these commitments more clearly. This one was called "On Physical Lines of Force." He began: "We are dissatisfied with the explanation founded on the hypothesis of attractive and repellant forces directed towards the magnetic poles, even though we may have satisfied ourselves that the phenomenon is in strict accordance with that hypothesis, and we cannot help thinking that in every place where we find these lines of force, some physical state or action must exist in sufficient energy to produce the actual phenomena [here, the patterns of iron filings around a magnet]."¹⁰

Maxwell proposed to try to explain electric forces as the result of strains in an elastic solid, the ether. He began by equating magnetic forces to pressures generated mechanically by "molecular vortices" that filled much of space. This was somewhat speculative but not implausible; such vortices had been posited by William Thomson to account for the rotation of polarized light in the neighborhood of magnets, an effect discovered by Faraday. But there was an obvious problem from the standpoint of mechanical realism: It was difficult to imagine these vortices all packed together, indeed filling space, and with adjacent regions of neighboring vortices moving in opposite directions. On this account, Maxwell introduced "idle wheels" separating the layers of ether vortices. He was not at all tempted to believe that anything like these wheels really existed. Thus his construction was an analogy and not a hypothesis. It proved to be a marvelously effective one, pointing toward the integration of electricity and magnetism. The most striking result was Maxwell's calculation that the tension in this ethereal medium would support waves that should travel at approximately the speed of light. This was a satisfying consequence of his model. Light, he concluded, must be nothing other than waves in the electromagnetic ether.

This last statement was no longer merely an analogy but a hypothesis. And this was typical of Maxwell's style, to reason by way of fictions in order to reach, in the end, a likely physical truth. On this account his analogies could not be chosen arbitrarily but had to be somehow like the truth, even if they were not strictly true. Although he posited mechanisms that were no more than "illustrative" they were also consistent with what Maxwell took to be a general truth, that energy "resides in the electromagnetic field, in the space surrounding the electrified and magnetic bodies, as well as in those bodies themselves."¹¹ Or more generally, he used mechanical models because he believed in the ultimate possibility of mechanical explanation. For Maxwell and his followers,

physical reality was mechanical. The highest purpose of models was to contribute to the discovery of physical truth. Successful prediction by itself was decidedly not the end of science.¹² Indeed, Maxwell's antipositivism went farther still, for he deemed it self-evident "that the laws of nature are not mere arbitrary and unconnected decisions of Supreme Power, but . . . form essential parts of one universal system, in which infinite Power serves only to reveal unsearchable wisdom and eternal Truth."¹³

It required great intellectual faith to give models so crucial a role in science. Even Maxwell felt the appeal of descriptionism as a counter to the profusion of models. He dropped the analogies and relied on mathematical deductions when he summed up his work in the authoritative *Treatise on Electricity and Magnetism* (1873). Particularly revealing is the reaction of William Thomson (later Lord Kelvin) to his work. Thomson, ironically, had provided the model for Maxwell's modeling. He had, for example, introduced the flow analogy to electricity more than a decade before Maxwell, in 1841. Thomson, though, could not countenance a form of argument in which a fictitious analogy supported a train of reasoning leading to predictions that had no basis in experimental fact. He refused to allow new physical entities to be hypothesized on the basis of a mere analogy. Maxwell's mathematical reasoning, it seemed, was supported by no more than analogical suppositions. It thus soared too far above the solid ground of measurement and experimental demonstration for his taste.¹⁴

To be sure, Thomson no more opposed mathematics than he did models. But he used them differently. He was brought up on the mathematical physics of Joseph Fourier. Fourier's theory of heat was, in a way that Auguste Comte appreciated, positivistic. Not that Fourier was uninterested in physical explanations. Neither did he lack well-formed views about the nature of heat. But his *Analytical Theory of Heat* (1822) set out from strictly phenomenalist assumptions about the macroscopic flow of heat, precisely in order to avoid the contested issue of whether it was a substance or a form of motion. Thomson relied heavily on Fourier's mathematics in most of his early research, and in fact the analogy in his 1841 study of electricity was to Fourier's formulation of the flow of heat. He also imbibed much of Fourier's philosophy of mathematical physics. Mathematics, setting out from secure experimental knowledge, could provide the enormously useful service of describing phenomena. Mathematical analogies could assist this effort. But one must use the analogies only to infer quantitative relations, not new entities. Like Fourier, then, Thomson was inclined to descriptionism in regard to mathematics; he believed that one could

use mathematics without committing to a physical theory. He was, on the other hand, the very opposite of a descriptionist in his physics, especially later in his career. "I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it." He meant a likeness, not a speculative analogy.¹⁵

Among Continental physicists, the urge to employ strictly phenomenological mathematics in order to avoid commitment to a physical theory was to a large degree a response to Maxwell. Maxwell used mechanical analogies in an extraordinarily fruitful way. Especially after Hertz demonstrated experimentally the existence of electromagnetic waves in 1887, everyone conceded this. But what was to be made of his ether vortices and idle wheels, analogies that even Maxwell considered mere aids to reasoning? Maxwell, moreover, was willing to rely on diverse, even inconsistent, analogies in pursuit of an adequate mathematical formulation. The mechanical view of nature remained plausible enough, but evidently the mechanical representations of electricity were undetermined by the phenomena. Perhaps, as Boltzmann clearly believed, mechanics could provide a continuing source of useful insights. But even Boltzmann conceded that truth could not be claimed for mere models. Evidently physicists who, like the mythical physician, aspired first of all to tell no lies ought in the end to dispense with these mechanical representations. Descriptionism offered the prospect of certainty, which mechanical reductions now seemed most unlikely to provide.

Probably the two most influential descriptionist attempts to reformulate physics were made by Gustav Kirchhoff beginning in 1876, and by Heinrich Hertz in the 1890s. Unlike much of Mach's work, these were written not mainly as critical accounts of existing or past physical theory but rather as systematic attempts to reconstruct mechanics to make it independent of all metaphysical concepts or physical hypotheses. Kirchhoff explicitly disavowed any intention to get at the real nature of things. Physics, he explained, can do no more than describe the regularities of phenomena. These descriptions should not go beyond what we can actually observe. Accordingly, Kirchhoff preferred to treat heat phenomenologically and not to assume an identity with molecular motion. Eventually, and somewhat reluctantly, he introduced the concept of the molecule, not as a really existing entity but rather as a useful hypothesis for deriving some of the laws of thermodynamics. He also refused to assume the real existence of forces, treating them instead as a convenient shorthand to describe the phenomena of motion. Hertz was more radical, arguing that the notion of force rests on a doubtful metaphysics and adds nothing to physics. An adequate mathematical

description ought to dispense with forces and talk instead of accelerations, which can be observed.

In a way this austere physics was notable for its modesty, its forbearance from claiming an understanding of nature itself. It was, however, not without its pretensions. By giving up metaphysics and relying instead on mathematical description, physics could eliminate whatever was doubtful and attain to almost perfect rigor and certainty. In principle, though of course often not in practice, physical theory was to become like pure mathematics.¹⁶ Mathematics itself was in these years turning increasingly away from its traditional claims to be about space, time, or mechanics and stressing instead a rigorous process of deduction from assumptions that could be chosen arbitrarily, provided they proved to be consistent.¹⁷ Kirchhoff and Hertz wrote in a resolutely mathematical way and sought to buy some of the certainty of mathematics at the cost of claims to a real understanding of causes and mechanisms. Typical is Hertz's famous assertion, reflecting his impatience with speculative mechanical model building, that Maxwell's theory of electromagnetism is nothing other than Maxwell's equations. By eschewing hypotheses, Hertz and Kirchhoff could aspire to full rigor and an almost timeless validity.¹⁸

Boltzmann, whose career was mainly devoted to statistical reasoning from molecular models, was strongly opposed to the descriptionist asceticism that could tolerate no models or hypotheses. "Hertz is right when he says: 'The rigour of science requires that we distinguish the undraped figure of nature itself from the gay-coloured vesture with which we clothe it at our pleasure.' But I think the predilection for nudity would be carried too far if we were to forgo every hypothesis."¹⁹ And he recognized that claims to rigorous scientific description were less modest than Kirchhoff made them appear. What, he asked, would Kirchhoff count as an explanation? "If one seeks to explain motions from forces and these from the nature of things, that is phenomena from things in themselves, one always seems to start from the view that explanation requires reducing the explanandum to some new principle external to it. This view is alien to natural science, which merely resolves complex things into components that are simpler but the same in kind, or reduces complicated laws to more fundamental ones." Many problems of physics, he explained, "are like the question once put to the painter, what picture he was hiding behind the curtain, to which he replied 'the curtain is the picture.' For when requested to deceive experts by his art, he had painted a picture representing a curtain. Is not perhaps the veil that conceals the nature of things from us

just like that painted curtain?"²⁰ For Boltzmann, descriptionism was compatible with a temperate realism. "The question whether matter is atomistically constituted or continuous . . . reduces to the question: Which represents the observed properties of matter most accurately. . . . ? Of course this does not answer the old philosophic question, but we are cured of the urge to want to decide it along a path that is devoid of sense and hope."²¹

Boltzmann's concessions to descriptionism were made originally in response to Kirchhoff, but increasingly, toward the end of his career, they grew out of his running dialogue and debate with Ernst Mach. Boltzmann assumed the posture of the practicing scientist in relation to Mach: Mach's epistemology was unobjectionable, provided it was never used to restrict scientific theorizing or scientific models but only to interpret science after the fact. It was, Boltzmann held, a matter of small consequence whether one called scientific knowledge true or only economical. Mach, in contrast, insisted that his philosophy had consequences, and he was sharply critical of prevailing theories of matter, space, and energy. For example, and most notoriously, he considered the doctrine of atomism a metaphysical excrescence on physics and held that scientists should avoid such concepts. This implied the illegitimacy of Boltzmann's molecular gas theory, his greatest achievement. Boltzmann was not alone among physicists in doubting that such positivist asceticism was healthy for science. By 1916, when Mach died, his star was clearly descending among physicists. But his motivations came largely from outside physics, and his reputation in the human sciences suffered no such decline.

MACH, MONISM, AND PSYCHOPHYSICS

Mach's philosophy has sometimes been taken as the exemplar of an exploitative, capitalistic mentality toward nature. He denied that we can gain true knowledge of the world, denied even that it is meaningful to talk about nature independently of our relations to it. Drawing inspiration from an economist, his Graz colleague Emmanuel Herrmann, he adopted as cornerstone of his philosophy the principle of economy. Scientific laws, he held, are valuable insofar as they permit us to economize on thought and thus to act more effectively. By compressing the lessons of experience into laws and generalizations, science compensates for the limitations of memory. Henri Poincaré compared the body of scientific knowledge to a well-ordered library catalogue, and one can imagine Mach expressing himself similarly. Mach explained this in Dar-

winian terms, valuing science mainly for its contribution to human survival, and disregarding nature except insofar as humans interact with it. "The biological task of science is to provide the fully developed human organism with as perfect a means of orientating himself as possible. No other scientific ideal can be realized, and any other must be meaningless."²² Science, evidently, should be considered a branch of engineering. Its ultimate product is not knowledge, but control.

Mach was read this way by Jacques Loeb, who found in Machian philosophy a justification for a biology aiming above all to gain experiential domination of life.²³ There are many statements in Mach's work which, taken in isolation, support such an interpretation. But it reflects at best a highly partial and partisan reading of Mach's work. To comprehend Mach adequately, we need especially to attend to his psychological thought, his attachment to Gustav Fechner's psychophysics. And this, in turn, leads us to Mach's attachment to a somewhat attenuated Romanicism and his rejection of a dualism of mind and matter. Mach denied that nature could be an object of human knowledge not because he viewed external nature as a thing lacking integrity and fit only for exploration but because he denied that a valid distinction could be made between subject and object. And for this reason he considered that scientific claims ought simultaneously to be about human actors and the world in which they act. Physics could not pertain exclusively to physical objects. A defensible philosophy of physics must by definition be valid also for psychology—at least for the psychology of sense observation.²⁴

The increasing emphasis in nineteenth-century physics on precise measurement created the conditions for an alliance between experimental physics and the psychology of perception. Stellar magnitudes, observational errors, and the "personal equation" were all interpreted in psychological terms. C. S. Peirce shared with Mach this combination of interests. But the alliance was undoubtedly strongest in the German-language tradition. Already in the year after receiving his doctorate in physics, 1861, Mach wrote to Fechner praising his book on psychophysics and claiming that he had himself long been occupied with mathematical psychology.²⁵ He concluded early that materialistic presuppositions would not get him far toward finding the exact laws of mental life. In 1863 he wrote: "We want to avoid all hypotheses [and] strike out on a rigorous path of experience, such as was trodden by the statisticians, especially Quételet."²⁶ These strictures applied equally to physics: "Any one who has in mind the gathering up of the sciences into a single whole, has to look for a conception to which he can hold in every department of science."²⁷

That conception, as Mach defined it, emphasized the common basis

of all knowledge in the immediacy of experience rather than, as many preferred, mechanical explanation. But we should not confuse what is commonly labeled Mach's "positivism" with a lack of interest in the nature of reality. His methodological claims were simultaneously ontological ones. His ontology, though, had no place for objects without subjects. Beside his claim that "a real economy of scientific thought cannot be attained by mechanical hypotheses," we find: "[The mechanical theory of nature . . . is an artificial conception." Or, most revealing: "Purely mechanical phenomena . . . are abstractions."²⁸ He meant that all physical phenomena are simultaneously psychological, that nature must properly be understood in terms of a psychophysical parallelism, a monism of the physical and mental.

Mach was, that is, a true member of the tradition of Fechner. Fechner had been an accomplished researcher on electricity before he began publishing on psychophysics. He interpreted even his physics in holistic, antireductionistic terms. Nor was he isolated, a mere crackpot. Among his allies were his Leipzig colleague and fellow psychophysicist Ernst Heinrich Weber, and Weber's brother Wilhelm. Wilhelm Weber provided the standard formulation of electrical and magnetic forces on the Continent until the vindication of Maxwell by Hertz in 1887. Like Fechner, he was concerned to define a physics that would also incorporate spirit, *Geist*, at every level.²⁹

Much of this was rather attenuated in Mach's writing. In contrast to Fechner, he was reluctant to talk of *Geist* as somehow inhabiting every physical object. Neither, though, did he accept the real existence of objects. This ostensible antirealism reflected a more radical holism, in which it was illegitimate to speak of matter without simultaneously recognizing the presence of consciousness. Of course this was not an objectified consciousness, out there, independent of the observer. But neither were there objectified atoms, or stones or gardens, independent of observers. Matter, he explained, is "a highly natural, unconsciously constructed mental symbol for a relatively stable complex of sensational elements." Mach's was an unanalyzable world in which there could be no barrier between physics and psychology, inside and outside. He aimed to build a "unified monistic structure," and so "get rid of the distressing confusions of dualism."³⁰

Mach's philosophy thus permitted neither subject nor object but only, and always, an interaction. Moreover, the world never holds still. It may be convenient to assume the continuity of objects through these changes, but we must recognize this assumption as a fiction. There is no thing in itself underlying appearances. In the same way, there is no stable subject. Mach argued that there is little reason to fear death, since it "occurs

in life in abundant measure." The self, ever changing, is hence always dying and always being born. Mach's slogan for this, "Das Ich ist unrettbar" ("the ego cannot be saved"), became a catch phrase and bought him a reputation in Viennese literary circles. In place of objects persisting through time, Mach recognized only a flux of experience.³¹

Sometimes Mach's conception of science sounds colorless and austere, even phlistine. Our thoughts should be ruled by an economy of usefulness. Mach, though, used his principle of economy mainly against what he regarded as impermissible attempts at reduction. Einstein was probably right when he emphasized the mainly negative function of Mach's philosophy: "It cannot give birth to anything living; it can only exterminate harmful vermin."³² The main object of Mach's economical scorn was the view that mechanical explanation is somehow deeper and truer than other kinds of understanding, or than concrete experience. Atoms, meaning invisible but unchanging particles treated as the building blocks of all reality and as more fundamental than concrete experience, were the objects of his special scorn, even though, as Boltzmann insisted, they had excellent credentials from the standpoint of economical explanation. Only a large and unsupported infusion of the a priori permitted him to argue abstractly against every attempt at mechanical reduction that it must be worse than useless, since it can at best aspire to substitute for known facts "an equally large number of hypotheses."³³

Opposition to materialism informed also his critique of the doctrine of energy conservation. He applauded the experimental discovery of conversion processes by which electricity or heat could be used to perform mechanical work in precisely measured ratios. But he considered it quite unjustifiable to surmise from these processes that heat and electricity are merely different embodiments of some fundamental substance, energy, especially when energy itself was understood as ultimately reducible to matter in motion. It required history, which in Mach's hands always performed a role akin to psychoanalysis, to explain why such a pathological conception had ever arisen. "Substance," he explained, is merely "a convenient word for a gap in our thoughts." All we have in science is "knowledge of the connexion of appearances with one another. What we represent to ourselves behind the appearances exists *only* in our understanding, and has for us only the value of a *memoria technica* or formula. . . . But if this way of presentation is so limited and inflexible that it no longer allows us to follow the many-sidedness of phenomena" it should be discarded.³⁴

Mach was thus austere about entities that purport to reduce or rigidify experience. The purpose, though, was to preserve the primacy of experience, to warn against faith in an independently existing world.

This applied to atoms, of course, but also, and equally, to space and time, both of which depend on human intuitions and cannot be properties of an external reality. From the study of psychophysics and Herbart's works, he explained, "I became convinced that the intuition of space is bound up with the organization of the senses, and, consequently, that we are not justified in ascribing spatial properties to things which are not perceived by the senses."³⁵

Whereas Mach wielded his razor unsparingly in excising everything that might appear uneconomical in mechanical representations, he was far more indulgent in regard to the experience of the senses. He refused to countenance material atoms but happily made elementary sensations into the solid foundation of science. We should "resolve the whole material world into elements which at the same time are also elements of the psychical world and, as such, are commonly called sensations; . . . we regard it as the sole task of science to inquire into the connexion and combination of these elements, which are of the same nature in all departments."³⁶ And his ideal of explanation was more nearly poetical than scientific, clearly valuing vividness over economy.

Our knowledge of a natural phenomenon, say of an earthquake, is as complete as possible when our thoughts so marshal before the eye of the mind all the relevant sense-given facts of the case that they may be regarded almost as a substitute for the phenomenon itself, and the facts appear to us as old familiar figures, having no power to occasion surprise. When, in imagination, we hear the subterranean thunders, feel the oscillation of the earth, figure to ourselves the sensation produced by the rising and sinking of the ground, the cracking of the walls, the falling of the plaster, the movement of the furniture and the pictures, the stopping of the clocks, the rattling and smashing of windows, the wrenching of the door-posts, the jamming of the doors; when we see in mind the oncoming undulation passing over a forest as lightly as a gust of wind over a field of grain, breaking the branches of the trees; when we see the town enveloped in a cloud of dust, hear the bells begin to ring in the towers; further, when the subterranean processes, which are at present unknown to us, shall stand out in full sensational reality before our eyes, so that we shall see the earthquake advancing as we see a wagon approaching in the distance till finally we hear the earth shaking beneath our feet,—then more insight than this we cannot have, and more we do not require.

Auxiliary conceptions such as mathematical formulas may be useful as aids to the mind, but they would be "devoid of value, could we

not reach, by their help, the graphic representation of the sense-given facts."³⁷

On occasion, he expressed doubt that such mental recanment of past or potential experience was attainable through science. "In the economical schematism of science lie both its strength and its weakness. Facts are always represented at a sacrifice of completeness and never with greater precision than fits the needs of the moment."³⁸ Typically, though, he represented science as an extension of normal reasoning about experience. His book *Knowledge and Error* is full of examples from ordinary life, and even cites numerous studies of animal behavior, to show that science is continuous with the mental life of men and animals. Forming hypotheses, for example, is natural, sometimes instinctual, and not limited to humans. The concepts of number and arithmetic have arisen out of ordinary human activities, such as trading, and more advanced forms of calculation are just mediated counting.³⁹ Where science departs from common experience, this is not necessarily an advantage. "Planned quantitative experiment yields many details, but our quantitative ideas educated by experiment gain their surest support if we relate them to those raw experiences."⁴⁰ Such abstract tools as formal or inductive logic are almost useless in reasoning from experience, for they presuppose identical conditions or events, and nothing in the world is ever repeated exactly.

Inferences, Mach argued, depend on judgments of similarity or analogy and are matters of psychology more than logic. He meant that they are somehow instinctive, dependent on the connectedness of self and nature rather than of detached, objective reasoning. In one essay he remarked on Bacon's view that experiment is the inquisition of nature, employing torture to extract its secrets. Mach was strongly opposed to such a domineering attitude. He displayed, as John Blackmore observes, "a Buddhist respect for the life and feelings of animals."⁴¹ And he argued repeatedly that "man, with all his thought and quests, is only a fragment of nature's life." If we understand this, we need not follow Bacon. "This view of nature, as of something designedly concealed from man, that can be unveiled only by force or dishonesty, chimed in better with the conceptions of the ancients than with modern notions."⁴² Precisely our intimacy with nature permits successful science. "If the ego is not a monad isolated from the world but a part of it, in the midst of a cosmic stream from which it has emerged and into which it is ready to dissolve back again, then we shall no longer be inclined to regard the world as an unknowable something. We are then close enough to ourselves and in sufficient affinity to other parts of the world to hope for real knowledge."⁴³

Mach has generally been read as a skeptic about the possibility of real human knowledge. Robert Musil, whose *Man without Qualities* reflects the Machian notion of the unsavable ego, while a doctoral student in psychology studied Mach and was troubled that his philosophy allowed "no truth at all in the authentic sense but only a practical convention contributing to self-preservation."⁴⁴ This rather misses the point, that our understanding is all the richer when we do not force the vivid world of experience into a strait jacket of objective things in themselves. We need posit no such misunderstanding to understand the fierce criticism issued by Max Planck in the same year, 1908. Planck was already something of a statesman of German science, and in that capacity he felt an obligation to combat dangerous heresies. The ideal of truly objective knowledge, Planck explained in his autobiography, was the beacon that drew him to science as a child. He found it "of paramount importance that the outside world is something independent from man, something absolute, and the quest for laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life."⁴⁵ In his essay on the "unity of the physical world picture," which contained his sharpest condemnation of Machian philosophy, he defended precisely this point of view. Thus his own radiation law seemed to him especially noteworthy because it revealed a new constant of nature. With such constants, one could define absolute units of length, time, mass, and temperature which would be valid "for all times and even for nonterrestrial and nonhuman cultures." Mach, he went on, fails utterly to comprehend the highest goal of science: "to free the physical world picture completely from the individuality of the spirit (*Geist*) that forms it."⁴⁶

From Mach's perspective, Planck was the sort of man who aspires to crawl out of his own skin, to scratch out his eyes because they get in the way of unmediated seeing. "Concern for a physics valid for all times and peoples including Martians seems to me very premature, even almost comic, while many everyday physical questions press upon us."⁴⁷ Mach was defended by Wilhelm Ostwald. One of the founders of physical chemistry, Ostwald was a severe opponent of Boltzmann and champion of a unified physics based on an irreducible concept of energy. Mach was lukewarm toward this energeticism but appreciated Ostwald's commitment to a unity that went far beyond physics. Ostwald provided much of the leadership for the German Monist League, and he regarded energy as simultaneously physical and psychological. Accordingly, he chided Planck for his pale conception of unity. A unified world picture, he argued, cannot be merely physical. Mach's ideas were greatly superior, for they applied simultaneously to the physical and psychical.⁴⁸

The drive for a union of physical and psychological was evident also in Mach's life as an experimental scientist. He pursued psychophysics not only as a reader but also as an experimenter. And perhaps on this account it is not surprising that his philosophy should have been especially appreciated by those who wanted the moral sciences (*Geisteswissenschaften*) raised to a higher level. Certainly this was central to the concerns of the Vienna Circle, which drew inspiration from Mach. Friedrich von Hayek reports that Mach's work was especially influential among students of the social sciences in Vienna when he was studying there, from 1918 to 1921. He explains that Mach provided the only alternative to the "orthodox" philosophy and that "the methodological or scientific character of their theories were much less secure than in the natural sciences, and on that account they wanted particularly to form a clear picture of what made a proper science."⁴⁹ Both Hayek's social scientists and the Vienna philosophers went well beyond Mach's own writings, for he never applied his own methodological strictures to the social or mental sciences.

We can easily imagine, though, why an aspiring economist would have found inspiration in a philosophy that denied the dependence of science on direct access to objective nature. A few years earlier, the Lausanne economist Léon Walras remarked about Henri Poincaré: "One of the masters of modern science . . . concluded that [physical] masses are the coefficients which are conveniently introduced into the calculations. Fine! This . . . encourages me to inquire as to whether all concepts, *mass* and *force* as much as *utility* and *raretés* might not simply be names given to hypothetical causes." It is both valid and essential to employ these causes in our calculations, so that economics can be worked out in a "strict and clear mathematical language."⁵⁰ Poincaré, who Walras sent this, responded skeptically, and Mach, too, could be interpreted this way only with a most liberal reading. Social scientists in the Anglo-American tradition, in contrast, required no imaginative readings to pick up this message. For it formed the core of Karl Pearson's neo-Machian *Grammar of Science*. With double irony, Mach's denial of real objects in physics was held up as the foundation for objective knowledge in the human sciences.

PEARSON, POSITIVISM, AND SCIENTIFIC METHOD

Pearson was exposed to the full range of descriptionist sources. He studied with Maxwell as an undergraduate at Cambridge in the late 1870s, then traveled to Germany and heard Kirchhoff's lectures in Berlin. He

became an admirer of Mach's writings and corresponded with him. He also immersed himself in German culture, or perhaps one should say *Kultur*, for he was inspired by his wanderings to use Karl rather than Carl as his Christian name. Many Englishmen in his day were drawn to the romanticist strains of German philosophy, but few more than Pearson. He wrote, very skillfully, on German religious history. He tried his hand at a romantic novel, *The New Werther*. And he became thoroughly antimaterialist, much after the manner of Mach. In an 1885 essay on "Matter and Soul," he argued that science has not materialized the world but idealized it by proving it to be intelligible. He held that the world can never be explained, only described, in terms of matter. Perhaps, he speculated, Schopenhauer was right to place will at the heart of things. Or, in a slightly different vein, who can tell whether matter is conscious? We find no such allusions to pampychism in Pearson's mature work, and in the reprint of this essay he editorialized: "The writer had not [yet] realized all science as description." But his devout antimaterialism lasted at least to the 1911 edition of his *Grammar of Science*. As with Mach, it was integral to his antirealism.⁵¹

Already, though, Pearson's philosophy of science revealed a new twist, a Saint-Simonian (or Comtean) or Young Hegelian strain to complement his Machian positivism. This was his emphasis on the relation of science to religion and to social order. The true basis of religion, he thought, may be the deification of the human mind and of its supremacy over matter. This would make scientists into high priests.⁵² Or religion may be the veneration not of the abstract humanity of positivism, but of the local group, the state. The social instinct in man, he explained, is the product of a long evolutionary struggle of group against group.⁵³ Pearson returned from Germany a socialist, by his own description. He even gave lectures on Marx. But he remained highly appreciative of the role of religion, provided it was not dogmatic. And indeed, he found this to be true of real historical religion, as in the case of medieval Germany. The German passion play reveals an admirable philosophy of life based not on the teachings of Jesus but on social values of morality, conscience, and community.⁵⁴ In general, he had much more use for religious institutions than for theistic religion itself. In a historical essay on Luther's "revolution," he lamented the terrible consequences of Luther's fanaticism. The Catholic church in 1500 was sufficiently tolerant to embrace Erasmus, Reuchlin, and Muth. Who can say that it would not have developed along with scientific culture, so that in 1880 it might well have had room for Matthew Arnold and T. H. Huxley? Its dogmas "gradually slipping into forgetfulness," the church "might possibly have become the universal instrument of moral

progress and mental culture." Then we would "now be enjoying the blessings of a universal church, embracing all that is best of the intellect of our time." Unfortunately, Luther's dogmatic appeals to the ignorant masses "dragged Europe into a flood of theological controversy, and forced the Church into a process of doctrinal crystallisation, from which it can now never recover." Alas, Luther never grasped the great "law of development," that progress never takes place by revolution and true reformation can come about only through a slow process of "genuine education."⁵⁵

Unlike Comte, Pearson experienced the most intense enthusiasm for the institutions and festivals of organized religion in his youth. Already in these writings from his 20s, though, we can find the crucial insight that would bind his social to his scientific philosophy. It is a longing for union and harmony, reaching even toward a monism of matter and spirit. Science, he explained, means first of all consensus. In later writings he would emphasize strongly that this agreement comes about through a shared method. In the early 1880s he put more stress on moral qualities among the scientists. In science, he explained, "there must be no interested motive, no working to support a party, an individual, or a theory; such action but leads to the distortion of knowledge, and those who do not seek truth from an unbiased standpoint, from the freethinker's standpoint, ministers in the devil's synagogue."⁵⁶ Science, then, provided the archetype for the *unrethbar Ich*, the disappearing subject, and on that account the scientist should be the model citizen. Science, like socialism, is "the subjection of all individual action to the welfare of society."⁵⁷

Pearson's descriptionism can in no way be understood as a sop to the old order. He was a tireless critic of the political elites of his own day. He stated in no uncertain terms that Christianity could no longer support their outdated morality. He aimed to create a new social hierarchy, and a new religion, based on scientific merit and scientific knowledge, with scientists elevated to "high priests of freethought."⁵⁸ And Pearson spoke openly and repeatedly in favor of radical (though gradual) social change, and creation in Britain of socialism. His philosophy made no concessions to old elites. Instead it embodied an aggressive campaign to expand the domain of science into religion and politics. By making science neutral with respect to subject matter, indeed by denying that it has a distinctive subject matter, he cleared the way to define it instead in terms of some universally valid method. Science "claims that the whole range of phenomena, mental as well as physical — the entire universe — is its field." "The field of science is unlimited; its material is endless, every group of natural phenomena, every phase of social life, every

stage of past or present development is material for science. *The unity of all science consists alone in its method, not in its material.*"⁵⁹ Not that scientists should fear to admit their ignorance, but any attempt to limit the possibilities of knowledge—Du Bois-Reymond's *Ignorabimus*—seemed to Pearson "a modesty which approaches despair," or worse, pseudoscience and bigotry.⁶⁰ By denying realism, by renouncing the object, he hoped to establish the conditions for a universal reign of disinterested objectivity.

By far Pearson's most influential work of philosophy was his *Grammar of Science*, first published in 1892. The book grew out of lectures he delivered as the Gresham Professor of Geometry in London, and about half of the book is devoted to physics. In those sections, Pearson developed an interpretation of mechanics, and especially of the laws of motion, that owed greatly to Kirchhoff and Hertz. He defined mass in terms of a ratio of accelerations, based on the Newtonian assumption that every action has an equal and opposite reaction. He called force "a convenient measure of motion, and not its cause." He held that the "mechanical determinism" of nature means nothing more than the predictability of motion and denied that this predictability could justify a "crude materialism." He was not particularly dogmatic about entities. At the turn of the century, when Mendelian genetics became a powerful rival to his own biometry, he earned the opprobrium of biologists by arguing partly on philosophical grounds against the existence of genes, but he remained quite willing to allow that the "conceptions" of atom and molecule may usefully "reduce the complexity of our description of phenomena." Indeed, he did not object to the use of apparently contradictory expressions in different disciplines, for the standard of efficiency is loose enough that each may be deemed valid in its own domain. Natural knowledge, then, could not be dictated by nature but must be subordinated to human ends. Pearson argued that space and time are modes of perception, not really existing objects. And "geometrical conceptions," too, cannot be defined apart from human observers. A circle, for example, is a limit of perceptual experience.⁶¹

This development of the philosophy of physics, though, amounted to a long epilogue. It was included partly as a model of scientific method in the field Pearson then knew best and partly as testimony from the most prestigious of the disciplines that scientific laws relate primarily to human perceptions and human convenience. Following the method of mechanics, humans can identify laws of phenomena in any domain at all. "The scientific method is one and the same in all branches, and that method is the method of all logically trained minds." It consists "in the careful and often laborious classification of

facts, in the comparison of their relationships and sequences, and finally in the discovery by aid of the disciplined imagination of a brief statement or *formula*, which in a few words resumes a wide range of facts. Such a formula . . . is termed a scientific law."⁶² This, of course, was anything but a license for scientific nihilism. The mind imposes laws on nature, but it does not impose them in whatever way it pleases. A scientific law "is something universally valid," Pearson declared. Clearly he was claiming something less than Planck did. Having neutralized nature in order to universalize method, Pearson evidently had weakened his basis for claiming that all observers should agree. But Pearson was not worried about Martians. Humans can be expected to reach consensus because of their shared "perceptive faculties." Dissenters reveal only the defects of those faculties; scientific law is "valid for all *normal* human beings."⁶³

As with Mach, Pearson's denial of objects had as a consequence the disintegration also of the subject. He repeated Mach's claims that we must construct our sense of our own bodies and that we are impermanent—we die many deaths in the course of what we call our lives.⁶⁴ But this was only a secondary concern. More important to Pearson is the way in which science cancels our private selves as it makes us citizens. "The scientific man has above all things to strive at self-elimination in his judgments, to provide an argument which is as true for each individual mind as for his own." Science leads to "sequences and laws admitting of no play-room for individual fancy."⁶⁵ The fight against subjectivity is of course not the same thing as the denial of the unitary self. Mach never pushed the idea of the *unrettbar Ich* in this direction. Yet these two conceptions were commonly allied, and Pearson's combination of them lay squarely in the positivist tradition as defined by Auguste Comte. The effect, in both cases, was to demonstrate the logical priority and moral superiority of society over individuals.⁶⁶

In Pearson's moral universe, it is not only scientists who should sacrifice individuality and personal judgment for the common good. His consistent adherence to method talk and his dismissal of objects had one further advantage upon which he placed great emphasis. Anybody could learn scientific habits of mind. Pearson recognized that science was becoming increasingly the business of specialists—though his own career defied that trend. Most people will not become professional scientists. But everybody can learn the scientific method. It requires no more than a good education. And they can learn to apply it to all phases of their lives. Thinking abstractly, Pearson proposed, we can see that the ideal citizen "would form a judgment free from personal bias." Science permits the ideal to become the real. "*Modern*

science, as training the mind to an exact and impartial analysis of facts, is an education specially fitted to promote sound citizenship."⁶⁷ Virtuous citizens, to be sure, would not on this account become autonomous, capable of deciding correctly on every question. Rather, they would learn to defer to their superiors, to an elite of scientists who could reach decisions on the basis of a full, and of course thoroughly impersonal, consideration of the facts. Pearson was an uncompromising elitist. There have been few stronger advocates of the prerogatives of an intellectual class.⁶⁸

Pearson's philosophy of science was a considerable success in its time. Its publication coincided with the emergence of the social science disciplines in the burgeoning American universities, and social scientists were among its most fervent admirers. The new social scientists were desperately eager to prove themselves worthy of the epithet *science*, but even the most fervent evolutionists did not want their disciplines to be swallowed up by biology or physics. The identification of science with method rather than subject matter suited them perfectly. Mach was read as supporting a unity of science that did not depend on reductionism. This was part of the reason for the intense interest of American psychologists in Mach's monism of mind and matter. But Mach was also read in a variety of disciplines as an authority on scientific method, so that his legacy was at least commensurable with Karl Pearson's. Mach was most often cited in disciplines, such as biology and psychology, which were becoming experimental, while Pearson's focus on observation and statistical analysis was especially welcomed by the applied social disciplines.⁶⁹ His emphasis on impersonality as the hallmark of science defined a social role for expertise that was ideally suited to American democracy. Social scientists could offer counsel not in the guise of wise, interested elites but as mere mouthpieces for a disembodied science. They could disarm suspicions that their advice was self-interested by intoning the phrase *scientific method*. Pearsonian philosophy was especially appealing to foundation officials who wished to sponsor interdisciplinary research that would have definite implications for policy. A generalized method would facilitate the collaboration of specialists while guaranteeing its disinterestedness. Robert Bannister writes that in the early 1920s the new Social Science Research Council practically institutionalized Pearson's *Grammar of Science*.⁷⁰ The modern social sciences in America derived from relations among foundations, universities, governments, and researchers as complex as those for biology or industrial technology.⁷¹ Pearsonian and Machian philosophy proved useful, even indispensable, for negotiating at the boundaries.

American champions of descriptionism in social science could and did cite Mach as well as Pearson.⁷² The institutionalization of *The Grammar of Science*, though, depended on an aspect of Pearson's philosophy that was wholly missing from Mach's. Mach had much to say about interpreting scientific results but very little about strategies for doing science. Pearson, in contrast, offered a specific program for new disciplines seeking to attain the status of science. That program was statistics. *The Grammar of Science* states clearly that all science is fundamentally statistical. Perfect homogeneity is to be found nowhere in nature; there is always an element of variability. What physicists call laws, then, are in fact approximations, or "correlations." Physicists, for various reasons, are often able to achieve very high correlations, meaning that their parameters explain most of the variation with which their ideal. Biologists and social scientists generally have to content themselves with lower correlations, partly on account of the nature of their data and partly because their theories are not yet so well developed. But this is a distinction only of degree, not of kind. "No phenomena are causal; all phenomena are contingent, and the problem before us is to measure the degree of this contingency."⁷³

Not coincidentally, Pearson knew something about measuring degrees of contingency. In 1888, Francis Galton published a method of correlation applicable to data of any kind. Pearson, who had long been skeptical on philosophical grounds of all attempts to reduce biology and social science to quantitative terms, was soon afterward converted to the statistical faith. Quantification, he decided, was a matter of precise description, and not of reduction at all. From about 1893 until his death, some forty years later, he devoted himself almost exclusively to the improvement and expansion of statistical methods. Galton's work, he determined, was epoch-making even in philosophical terms, for it demonstrated that causation is no more than the limiting case of correlation.⁷⁴ Pearson himself first derived the standard formula for calculating correlation coefficients in 1896.

Statistics was ideally suited to Pearson's philosophical and political aims, and also to his own research talents. An applied mathematician by training, he proved exceptionally adept at developing techniques and showing how they could be applied to data in almost every discipline. Soon his methods became standard in many of these disciplines. They brought with them a heightened emphasis on the ideal of impersonality, for they were designed to make the drawing of inferences from data less a matter of informal judgment, more a straightforward application of mathematical rules. Pearson's statistics, like his philosophy, embodied his moral ideal of science and of citizenship, in which indi-

viduality is sacrificed for the public good. In another way, too, statistics reduced the importance of individuals. The new journal *Biometrika* proclaimed in 1901, in words that bear Pearson's stamp: "It is almost impossible to study any type of life without being impressed by the small importance of the individual. . . . Evolution must depend upon substantial changes in considerable numbers and its theory therefore belongs to that class of phenomena which statisticians have grown accustomed to refer to as mass-phenomena."⁷⁵

Pearson's statistical philosophy was equally disrespectful of objects and subjects. His defiance of boundaries extended even to the human skin, as is attested by his prominent advocacy of eugenics. And in general, a political vision underlay Pearson's whole philosophy, even his whole career. The relation of the state to its citizens was recapitulated by the relation of science to its objects. Because the world has no independent existence, it is quite incapable of resisting our methods of investigation. Pearson's antirealism implied that science knows no limits and hence that human communities could free themselves from the tyranny of arbitrary or self-interested opinion. For more than one reason, the individual is *unrettbar*.

CONCLUSION: PHYSICS AND POLITICAL CULTURE

It should be clear from this essay that late-nineteenth-century philosophy of physics resonated in important ways with the wider culture. Clearly, though, there is nothing so simple involved here as an influence of physics on culture. For Mach and Pearson—and one might well add Charles Sanders Peirce here, though he was sharply critical of *The Grammar of Science*—the philosophy of physics necessarily included problems of mind and matter, knowledge and community. This was not limited to the community of science but reached out into the broader political culture. The road from descriptionist philosophy of physics to the idealization of public knowledge as a basis for political order was anything but direct or inevitable. Boltzmann and Mach debated for years; Pearson drew political implications from their descriptionist move which neither would have accepted. And yet its most important and lasting implications were of a broadly political character. Even Mach spoke passionately though obscurely of the need to "start collaborating eagerly in realizing the ideal of a moral world order, with the help of our psychological and sociological insights."⁷⁶ Most interpreters have emphasized that the new philosophy of science of the late nineteenth century reflected a spirit of moderation,

a pragmatic temper. It has been associated with a less dogmatic form of expertise, and hence with a greater respect for democracy.⁷⁷ Descriptionist philosophy was characterized above all by a respect for surfaces, for what, in principle, is visible to anyone who cares to look. Yaron Ezrahi has identified this aspect of science as crucial in explaining its authority in the democratic culture of America. Michael Oakeshott's disdain for scientific experts expresses the point eloquently. In a cultivated society, he argued, the rationalist is like "a foreigner or man out of his social class, . . . bewildered by a tradition and a habit of behavior of which he knows only the surface; a butler or an observant housemaid has the advantage of him."⁷⁸ If science flourishes, in Oakeshott's view, it is because the political order knows no depth, because the leveling tendencies of modern democrats and technical experts have destroyed the picture and left only a screen.

From another standpoint, though, this willingness to be content with surfaces was anything but modest, either in its intentions or its effects. By denying the autonomy of objects, and by associating science with appearances, philosophers such as Pearson and, in a different way, Mach supported the expansion of science beyond all limits. Moreover, the sacrifice of depth was calculated to yield an increase of certainty. Ironically, science without objects left less room for the play of subjectivity. In this way it captured an ideal held dear even by many realists. If science inspired by descriptionist philosophy promoted democracy, this was by contributing to a political system of impersonal bureaucratic rules, not by modestly supplying facts and interpretations that would enrich public debate.⁷⁹