First exercise on decision theoretic notions
Risk and expected utility

Tamas can plant either wheat or corn in his field. If the spring is dry, then Tamas would make €2400 if he plants wheat, and €1200 if he plants corn. If the spring is rainy, then Tamas would make €1800 if he plants wheat, and €4200 if he plants corn.

At the national radio, the forecast said that there is a probability of 2/3 that the spring will be dry and 1/3 that the spring will be rainy.

Tamas has a total faith in the predictions of the national radio and fully believes their weather forecast.

Tamas is a fully rational agent. What will he do if he is risk neutral: plant wheat or plant corn (assume that he has to choose between these two options only, or that all other options are dominated)? What will he do if he is risk averse? Explain your answer with the theory of expected utility.

Reminder: someone is risk neutral when the marginal utility remains constant, he is risk averse when the marginal utility decreases and risk seeking when the marginal utility increases.

An agent is risk-averse with regard to some gains if he has decreased marginal utility for the gain in question. The marginal utility is the gain in utility obtained from getting more items: $u(n+m) - u(n)$. Thus, a decreasing marginal utility can be expressed as:

$$\text{For all } x, y \text{ and } m, \text{ if } x < y \text{ then } u(y+m) - u(y) < u(x+m) - u(x)$$

It says that the more you have of something (expressed with $y > x$), the smaller the utility you get from getting m more of it (expressed with the second part of the above proposition: $u(y+m) - u(y) < u(x+m) - u(x)$).

Figure 1 provides illustrations of cases of utility of money with increasing, decreasing and constant marginal utility. In the graphs, the utility of 0 unit of money is 0 and the utility of 100 unit of money is 100. This can be fixed arbitrarily but the other points are revealing of attitudes towards risk.

Imagine that Kinga has to choose between 50 items for sure or 0 items with probability 1/2 and 100 with probability 1/2. We have the following possibilities:

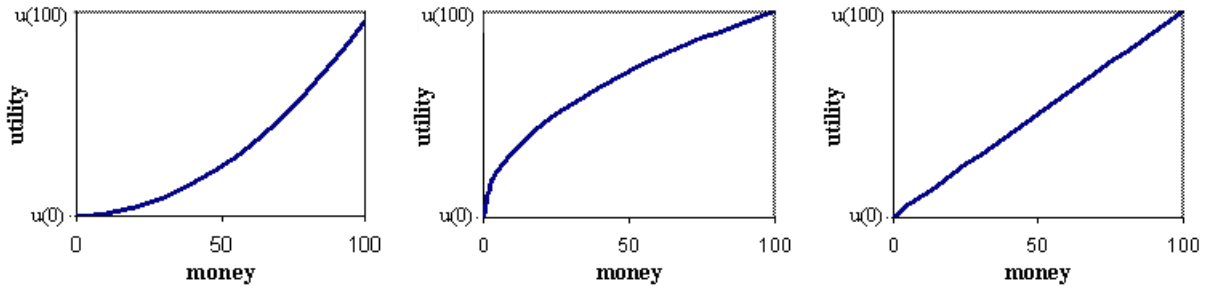

Figure 1: From left to right: risk seeking, risk averse, and risk neutral attitudes

- If Kinga is indifferent between the two, then her utility for 100 items is twice her utility for 50 items.

$$1/2 \times u(100) = u(50)$$
$$\text{therefore,} \quad u(50) = 50$$

  Thus Kinga's utility for 50 fits the function depicted on the right hand side graph. She's risk neutral.

- If Kinga prefers the sure option, then

$$1/2 \times u(100) < u(50)$$
$$\text{therefore,} \quad 50 < u(50)$$

  Thus Kinga's utility for 50 fits the function depicted in the middle graph of figure 1. She's risk averse.

- If Kinga prefers to take the bet, then

$$1/2 \times u(100) > u(50)$$
$$\text{therefore,} \quad 50 > u(50)$$

  Thus Kinga's utility for 50 fits the function depicted in the left-hand graph of figure 1. She's risk seeking.

Most people are risk averse for most things–including money. An interesting empirical result, however, is that while people are risk averse with regard to gains, they are risk seeking with regard to loss: they tend to prefer a bet where they keep what they have at a risk with probability p of loosing p items to a sure loss of one items. This gives the utility curve of figure 2 of Twersky and Kahneman's *prospect theory*.

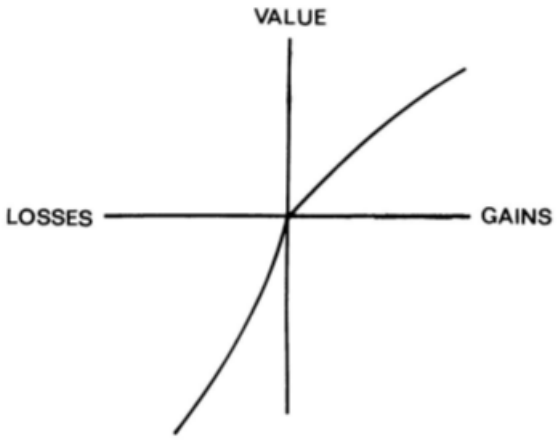

Figure 2: From Twersky and Kahneman's 1979 paper on Prospect Theory