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The Laws of Human Relations
and the Rules of Human Action Derived Therefrom

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1 The General Laws of Pleasure and Economic Value

Man wants to enjoy life and makes it his goal to increase pleasures enjoyed throughout life to the highest possible level.[1] Now, on the one hand, the life of a human being covers a considerable time span, and there are a large number of pleasures in life that man can obtain immediately; yet these pleasures have the consequence of imposing later, disproportionate deprivations. On the other hand, the most elevated, the purest pleasures become comprehensible, become real pleasures, only after man has educated himself for their appreciation. Therefore a man would be greatly mistaken if he thought to achieve most completely the purpose of his life by striving to obtain at every moment the pleasure that momentarily seems to him the greatest, without regard for its consequences. In order to measure the true magnitude of a specific pleasure, not only must account be taken of its magnitude, but also all the sacrifices imposed by its enjoyment must be subtracted from it. In particular, one must consider to what extent an enjoyment creates an obstacle to reaching both the physical as well as intellectual development that alone prepares man for higher and more refined pleasures. In other words:

Enjoyment must be so arranged that the total life pleasure should become a maximum.[2]

We find, therefore, that all men without exception act according to this principle from the cradle to the grave, the king as well as the beggar, the frivolous man of the world as well as the penitent monk; and if nevertheless, the actions of people seem so extraordinarily varied, this is solely the result of their different views as to the magnitudes of different pleasures. These magnitudes undoubtedly vary with educational level and also with the obstacles that enjoyment might create for future enjoyments. All men, agree, however, that everybody wants to maximize his pleasure throughout life. Even the ascetic, who apparently removes himself farthest from this purpose, demonstrates its fundamental truth when he thinks to attain heaven through self-castigation and deliberately chosen privations of all sorts. For quite apart from the fact that up to a point he derives satisfaction from this way of life, he is driven to this behavior only by the conviction that for all the privations willfully accepted here, he will be rewarded many, many times over in a life hereafter; should he be robbed of this conviction, he will immediately act quite the contrary. History furnishes abundant examples of *bons vivants* who have become ascetics and also of the opposite, of penitent monks who have turned into worldly *bons vivants*.

As far as the basic law is concerned, the ascetic differs from the *bon vivant* solely in that he is a far more insatiable egoist. What the earth has to offer does not suffice for him as total pleasure; he wants to have more, and he thinks that he can obtain this by his conduct. That men indeed frequently go to an extreme when they change to their earlier behavior demonstrates the general truth of this law most clearly, for they prove thereby that the impetus for change is felt very strongly, that is, that the force inducing this change in behavior is very decidedly present. Thus it is natural that a man should go from one extreme to the other just as soon as his conviction changes because of some circumstances, so that this force becomes directed toward a new aim. All positive religions of which we have historical knowledge regard this law as an axiom so incontestably established that they consider it even superfluous to proclaim its existence explicitly when they endeavor, by the promise of punishments and rewards in an afterlife, to keep men on the path that appears right to them. They attribute to these punishments and rewards an eternal duration in order to make it absolutely clear that the total life pleasure to be derived from following the right path will truly become a maximum.

Not only is this maximization viewed by all men without exception as life's ultimate purpose, but it also is undoubtedly the real purpose of man's life, willed by his Creator. We can explain the ineradicable and unceasing human desire to reach this objective only by the same procedure by which we seek to explain all other phenomena in nature, namely, by the assumption of appropriate forces working according to certain laws. Thus we must assume that the Creator has instilled in man a force from which the existence of this desire follows. To be sure, the real essence of any force can be known only by its effect and precisely in relation to its intensity. The purpose of this force, and hence the purpose of the Creator when He brought it into existence, could not have been other than that He wanted this effect and wanted men to act according to this force. Hence it would frustrate totally or in part the purpose of the Creator were we to attempt to neutralize this force in total or in part, as is the intention of some moral codes promulgated by men. How can a creature be so arrogant as to want to frustrate totally or partly the purposes of his Creator!

But such codes are the product not of presumption but of confusion. Indeed, people did not realize that it was most important to maximize the total life pleasure. At the same time, the harmful consequences of many enjoyments were perceived. It was thus concluded that these consequences

were inseparably connected with the enjoyment of life, and it was also believed, therefore, that enjoyment, being harmful, had to be prohibited. And people even went so far as to view enjoyment as something illicit per se.

Only a lack of understanding of those eternal and immutable laws by which the Creator rules His world is responsible for such unnatural prescriptions, which run counter to these fundamental laws. Only by exploring these laws can man protect himself from similar confusions.

From this purpose of life there follows the one and, therefore, chief rule for human conduct:

[L] *Man should organize his life so that his total life pleasure becomes a maximum.*

And the Creator, by establishing the force that produces in man the ineradicable and unceasing desire to reach this end, has made absolutely sure that man will reach this end once he himself has recognized the path that will lead him to it. And still more than that, immeasurably more! By subjecting the effectiveness of this force, like all other forces, to its own laws, He achieved for the social life of man exactly the same goal that He has accomplished for the planets through gravitation and its laws. In the same way that He established order among His worlds, He has established order among His human beings. As He has forever and immutably predetermined the paths of the planets by the laws of gravitation, He has predetermined for all eternity and invariably for all men the pattern of their social existence by the laws governing their power of enjoyment. In this way, He made sure that once man comprehends the laws pertaining to the operation of this force, *every individual concerned exclusively with his own personal welfare must bend his efforts to the benefit of all men in a manner that is best for the welfare of all mankind.* This, therefore, is the force that holds human society together; it is the bond that ties all men and forces them by advancing their own welfare through mutual exchange to further at the same time the welfare of others. And yet while this force renders incalculable benefits to mankind, it was so misunderstood that it was disparaged as unbridled desire for pleasure because it is liable to occasional misuse. For this reason, some fancied it a great merit to think that they had succeeded in suppressing it entirely or partially. This shows how far man can go astray when he fails to observe the revelations of the Creator, as He manifests them eternally, unchangeably, and uninterrupt-

edly in His creation, and when he takes instead human rules as his guideline! But here again is revealed the mysterious wisdom of the Creator. He must have anticipated this erring too, and for this reason He gave this force such an extraordinary strength that all human resistance to it can only weaken but not paralyze it. And no matter how man may try to suppress this force in one of its manifestations, it will always reappear with increased strength in an unexpected and unforeseen new manifestation. May we, therefore, never again lose sight of the commandment of the Creator, which He reveals to us daily in a thousand different forms over and over again with unmistakable signs! It is,

Man! Explore the laws of My creation and act in accordance with these laws!

For its genuineness or truth, this revelation needs no human testimony; it confirms itself in such an indubitable manner that any other proof seems superfluous.

In accordance, then, with this principle, an attempt must first be made to explore the laws according to which the force of enjoyment operates. Upon closer examination of the process of enjoyment, one finds the following common characteristics of all acts of enjoyment:

[A.1] *The magnitude* [*intensity*][3] *of pleasure decreases continuously if we continue to satisfy*[4] *one and the same enjoyment without interruption until satiety is ultimately reached.*

[A.2] *A similar decrease of the magnitude* [*intensity*] *takes place if we repeat a previously experienced pleasure. Not only does the initial magnitude* [*intensity*] *of the pleasure become smaller, but also the duration of the pleasure shortens, so that satiety is reached sooner. Moreover, the sooner the repetition, the smaller the initial magnitude* [*intensity*] *and the shorter the duration of the pleasure.*

Daily life offers thousands of instances confirming both of these attributes. The [magnitude of] pleasure of the artist offered the enjoyment of a new work of art will be the greatest at the moment when he has contemplated it long enough to take in all its details completely. This pleasure will constantly diminish, and depending on the object and the individual, after a shorter or longer time, the artist will tire. Satiety will be reached even though he should still be inclined to partake in other pleasures, and even if he should still be ready to enjoy other works of art of a similar

nature. Later, after a shorter or longer period—varying again with the object and the individual—the desire to repeat the same enjoyment occurs; because of his earlier acquaintance with the work of art, he will reach the peak sooner, but this peak will be that much more below that of the first act of enjoyment, the more frequently and the sooner the repetition has taken place. Also, with repeated enjoyments of the work of art, continued contemplation will again result in continued diminution of pleasure; satiety will again occur the earlier, the more often and the sooner the repetition has occurred. That repetition brings about an actual decrease of the peak of pleasure and a shortening of the duration of enjoyment becomes more evident with the increased frequency of repetition. The owner of a work of art, though he be the greatest art enthusiast, will gradually become more and more indifferent to the enjoyment of that piece of art. Eventually, whole days will pass when, being by himself, he will not seek the enjoyment of the work of art, and these interludes between the repeated enjoyments will become greater; on the other hand, the period of actual enjoyment will become shorter. This is the most obvious proof that the interest in that pleasure, that is, its magnitude, diminishes the more, the more frequently the enjoyment is repeated.

To reflect upon a certain subject, be this done by a person in isolation or when stimulated through conversation, will be increasingly absorbing for that person until he has comprehended the subject completely. Who does not remember the pleasure he has derived from the discovery, real or fancied, of a new truth! Subsequently, some pleasure is derived from dwelling on the subject for a while; but this diminishes more and more until in the end any further contemplation of the topic results in boredom. The repeated exploration of one and the same topic generates with each repetition a reduced pleasure, the more often and the more rapidly the repetition takes place. The sharing of this topic with others, which initially gives pleasure, is eventually transformed into teaching and becomes a chore.

And such a decrease of pleasure takes place for not only so-called intellectual pleasures but also, according to similar laws, material pleasures. To the man who allays his hunger with one single dish, the first mouthful tastes best; the second does not taste quite as good; the third, even less; and so on until, when he has nearly reached satiety, he is almost indifferent as to whether he takes the last bite. Experience confirms beyond doubt that repeated satiation with the same fare causes a decrease

of pleasure and a reduction in the quantity of the enjoyable consumption similar to the contraction of the period of intellectual pleasure. The poor man who has a roast only on holidays undoubtedly derives greater pleasure from eating his roast than one who has his daily fill of it; but for the latter, the pleasure afforded by eating all the roast he wants is all the more increased, the longer this enjoyment is delayed.

It hardly needs mentioning that this decrease of pleasure brought about by repeated enjoyment of one and the same object is not the same for all persons; how widely this phenomenon is recognized is further proved by often heard remarks of the following sort: Yes, I like to see, to taste, or, in general, to enjoy this or that, once or a few times; but more often than that I would not want it. It is confirmed in another way by the surprise caused us when we come across a good—such as bread—for which a decrease of the initial enjoyment is hardly observable even though the enjoyment is frequently repeated. This surprise can only be caused by the fact that we are accustomed to observing more drastic diminutions. The decrease is similarly conveyed by the common remark that initial enjoyment, or even its first or second repetition, justifies the amount of time and effort expended on it. One feels, however, that a longer duration or a more frequent repetition would cause such a reduction that the pleasure would no longer be sufficiently large relative to the time and effort spent. To show for all objects, as for the previously mentioned case of bread, that repetition, and repetition alone, causes a decrease, one need only cite the increased pleasure in eating bread for one who, used to obtaining it daily, if is deprived of this enjoyment for a few days. One could wager that a person cannot eat one and the same dish as his only food for many days in succession, even if, under other circumstances, the dish may be a delicacy for him, nor consume every day the same quantity. The basis of the bet is, of course, that the pleasure diminishes upon repetition and that, consequently, consumption also decreases.

This decline of pleasure resulting from continuous and repeated enjoyment of the same object should not be confused with the increase that anyone can achieve through the exercise of the senses of enjoyment. Exercise of the eye, ear, taste, and mind increases, in general, the enjoyment of the objects serving these senses; but continued and repeated enjoyment of one and the same object is subject, nevetheless, to the process of diminution.

The law of the decrease of the magnitude of pleasure thus applies with-

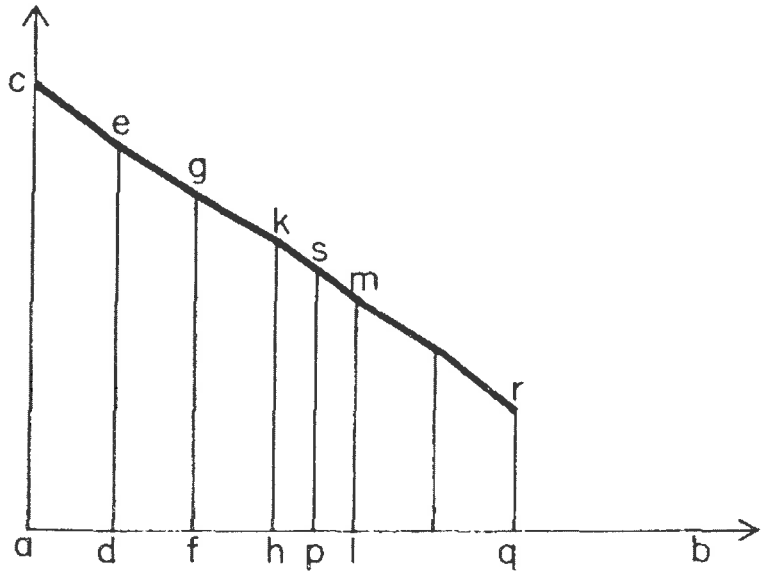

Figure 1.1

out exception to all pleasures, intellectual as well as material. *Just because the Creator made the capacity to enjoy, the desire for enjoyment, subject to this law, He made it possible for this desire to bring about such results as have been suggested in some detail.*

The incalculable importance of this law makes it desirable to obtain the clearest possible notion of it. A geometric diagram may be of help here. Whenever it is possible to represent by such a diagram a certain truth, we obtain a double advantage. First, the truth is presented not only for our reasoning power but also for another sense, namely, that of the eye, so that one obtains an image of the truth in the proper sense of the word. Second, if the diagram is correct, calculations can be more easily applied to it, and we can thus obtain for the result the guarantee of correctness that mathematics is able to establish. It is well-known that physicists appreciate this advantage very highly.

In our case, such a diagram can be drawn in the following manner: Let the time during which a pleasure lasts be presented by line *ab* (figure 1.1), so that any point on the line *ab* represents a corresponding instant of this time; hence every segment of the line represents a corresponding time interval. In this case, therefore, $\overline{ad}$, the first tenth, corresponds to the first tenth of the time period; $\overline{df}$, the second tenth, corresponds to the second tenth of the time period; and so on.[5] One may then imagine a perpendicular line erected at each point of the line *ab*, as shown, for example, for *a*, *d*, *f*, ... Let the height of each of these perpendicular lines be propor-

tional to the [intensity of] pleasure experienced at the corresponding instant of time. If one connects the end points of the verticals—here c, e, g, k, . . .—it becomes obvious that the areas $\overline{adec}$, $\overline{dfge}$, $\overline{fhkg}$, . . . represent exactly the relative magnitudes of pleasures in the intervals of time $\overline{ad}$, $\overline{df}$, $\overline{fh}$, . . . and that, in general, any area limited by two perpendiculars on the line ab and also limited by the lines cr and ab—say, $\overline{pqrs}$—corresponds to the time interval $\overline{pq}$ and represents the corresponding magnitude of the pleasure.

For the true presentation of such a picture for any actual pleasure, it would be obviously necessary to measure the magnitude [intensity] of pleasure for each and every instant of time, a problem for which no solution has been found so far, or, rather, no solution with the purpose clearly understood has been attempted. Now, in geometry, it is necessary to measure the actual dimensions of a given space in the real world in order to present its picture accurately. But such measuring is not required in order to elicit the laws of geometry since for this purpose it suffices to find, from the specific characteristics of space, means by which its parts can be mutually related. Similarly, in order to develop the laws governing enjoyment, it is not necessary to measure actually magnitudes of pleasure. Here, too, it suffices to develop the possibilities that may occur in enjoyment and place them in mutual relation. And just as in geometry the established laws subsequently open up the possibility of spatial measurements in cases where direct measurement would be forever impossible—I may recall the measurements of the astronomers—so in our case, too, the laws established will enable us later on to make measurements in the realm of enjoyment, where no means have been found so far to do so directly.

The laws just expounded concerning the decrease of the magnitude [intensity] of pleasure in cases of continued and repeated enjoyment yield sufficient data to discover these possibilities. They show us that in the case of a specific pleasure, the line cr (figure 1.1), whatever may be its particular shape, if followed from c to r, must move continuously and without interruption closer and closer to the line ab. For it is precisely this convergence, that is, the shortening of the perpendiculars on ab, that expresses geometrically the decrease of pleasure through continued enjoyment. This shows, further, that both lines have to converge to b, the point in time of temporary satiety, because when satiety is reached there, the length of its perpendicular is equal to zero. Direct observation, however,

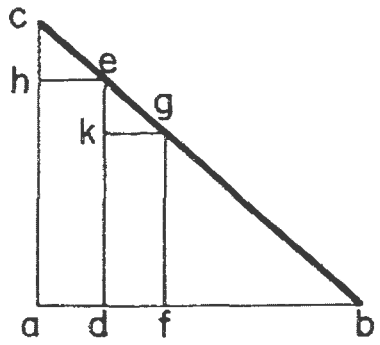

Figure 1.2

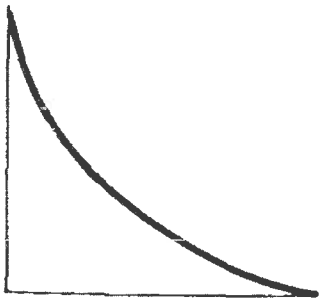

Figure 1.3

yields no clue whether figure 1.2, 1.3, 1.4, or perhaps 1.5 is more appropriate; this issue must remain undecided at this time. This being so, I choose provisionally, because of its greater simplicity of shape, figure 1.2, which is a rectilinear triangle. This figure satisfies the only condition that we have so far discovered, namely, the condition of the continuous decrease of the magnitude [intensity] of pleasure. It satisfies this condition in the manner, implicitly assumed, that the decrease is constant; that is, it is the same for equal time intervals. For a rectilinear triangle (figure 1.2), if $\overline{ad} = \overline{df}$, and eh and gk are parallel to ab, then $\overline{ch} = \overline{ac} - \overline{de} = \overline{ek} = \overline{de} - \overline{fg}$. We may, therefore, use this picture provisionally until the law of the decreasing intensity is found to be different—which, in turn, would cause the results to be modified and would also indicate just what these modifications should be.

Whatever the particular shape of the line cb in the graphical presentation of pleasure, it follows from the law of the decrease of the magnitude of pleasure by repeated enjoyment—when $\overline{abc}$ (figure 1.5) represents the magnitude of pleasure—that a specific sequence of repetition, undertaken more frequently than in the original situation, will cause $\overline{ab'}$ to be neces-

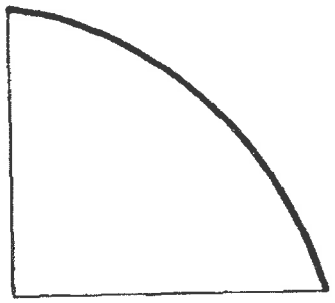

Figure 1.4

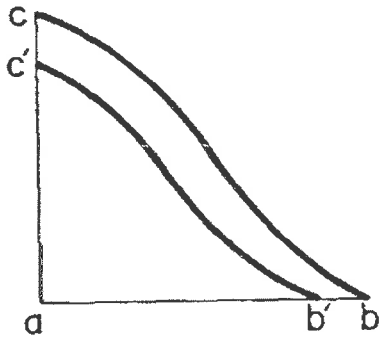

Figure 1.5

sarily smaller than $\overline{ab}$ and $\overline{ac'}$ also to be smaller than $\overline{ac}$. Furthermore, the specific shape of the line $c'b'$ will necessarily be similar to that of $\overline{bc}$; thus the perpendiculars in any given point of the line $\overline{ab}$ will intersect $\overline{b'c'}$ before $\overline{bc}$. From this condition it follows that the decrease of the magnitude [intensity] of pleasure is the result of natural forces that distinguish themselves only by the different intensities of their effects. With still more frequent repetition, the picture must become continuously smaller until it is reduced in the end to the single point a on the line $\overline{ab}$. In the case of all pleasures, this reduction to the single point a on the line $\overline{ab}$ must come about when the repetition takes place with such frequency that the next enjoyment begins just as soon as the pleasure of the previous enjoyment has been reduced to zero. In other words, this is a case when an uninterrupted enjoyment of one and the same pleasure is provided.

We find ourselves in this situation with pleasures such as nature yields us without our own effort—for example, with light, heat, the enjoyment of a landscape, and so forth. As a rule, we do not regard as a pleasure the light of day as such, apart from the object that we can enjoy with its help. Only if, due to some accident, uninterrupted enjoyment ceases, will the

pleasure be felt again, and this the more strongly, the longer we had to forgo it. After a long wakeful dark night, one greets the light of day for its own sake, apart from everything else, with genuine joy. Who does not recall prisoners' descriptions of the joyful excitement of their feelings when they saw once more the light of day after having been incarcerated in a dark jail! An uninterrupted stay in a room with a temperature perfectly agreeable to the body is not felt as pleasure, but entering such a room after having been previously exposed to extreme heat or cold produces enjoyment. The farmer who plows his field from early morning until late at night in the most magnificent environment does not find pleasure in viewing the scenery for whose enjoyment other people, however, are willing to undertake long journeys. Yet a short removal from his home grounds will increase the farmer's enjoyment to the extent of making him homesick.

From what has been said so far about the details of enjoyment, the following three theorems result:

[1.1] *With each specific pleasure, there is one definite manner of enjoying it, determined chiefly by the frequency of the repetition of that enjoyment, that will lead to a maximum of pleasure. Once this maximum is attained, the total pleasure decreases with either more or less frequent repetition.*

Examination of figure 1.5 makes this immediately clear. Once the magnitude [intensity] of pleasure at *b* has fallen to zero, the continuation of the enjoyment cannot increase the area $\overline{abc}$ representing the [magnitude of] total pleasure because the ordinates remain zero. Although in this instance an increase of the total pleasure cannot be achieved by an uninterrupted continuation, it could obviously be accomplished by interrupting for some time the availability of pleasure, for in this way the effect of the external world will be felt once more as pleasure. That a maximum exists can be clearly deduced from the fact that continued reduction in the frequency of satisfaction results in the end in a situation where the pleasure becomes available one time only or not at all; consequently, while less frequent satisfactions at first produce an increase in this sum total of enjoyment, subsequently this results in a decrease of this sum to zero. Within this range there must, therefore, necessarily be a mode of satisfaction allowing this sum to reach a maximum.

The fact that this law has been misunderstood so frequently has at all times brought to a large number of men the greatest possible misfortune,

that of missing the purpose of life. This is especially true for those who by conventional human standards seem to be preordained to achieve the maximum of human bliss, the so-called great personages of the world. To cite only the most glaring example of modern history, recall Louis XV, king of France. His retainers and courtesans, by dissipating the resources of an entire nation, succeeded in arranging the life of his court in such a fashion that everything that could have produced pleasure in a man of his level of physical and mental development was offered to him almost without interruption. The closer this aim was approximated, the more the total life pleasure was reduced for the pitiable Louis because in his case the highest point for each pleasure had, of course, been passed long ago. The consequence was that in the end even Madame Pompadour, who would not shy away from the most unnatural diversion as long as it promised to give some pleasure to Louis, did not succeed in driving away the most deadly boredom. Thus the failure to comprehend the previously mentioned law had only the following consequence: An entire nation was plunged into misery in order for Louis himself to become more miserable than the most repressed serf of his vast kingdom.

How enjoyment must be arranged for each single pleasure so that the maximum of pleasure shall be achieved is a question of fact. The answer depends on a more complete determination of the law of the decreasing magnitude [intensity] of pleasure, which, in turn, depends on the actual measure of pleasure. A specific answer cannot, therefore, be attempted at this time. Here it suffices to arrive at a general knowledge of the existence of this law and to know that a maximum exists for every pleasure and that it is determined, above all, by the greator frequency of repetition.

[1.2] *In order to maximize his total pleasure, an individual free to choose between several pleasures but whose time is not sufficient to enjoy all to satiety must proceed as follows: However different the absolute magnitudes of the various pleasures might be, before enjoying the greatest pleasure to satiety he must satisfy first all pleasures in part in such a manner that the magnitude [intensity] of each single pleasure at the moment when its enjoyment is broken off shall be the same for all pleasures.*[6]

All this follows from the law of the decreasing [intensity of] pleasure; we can make this clear with the aid of figure 1.6. Let *abc* represent the diagram of the magnitude of pleasure *A*, and $a'b'c'$, that of a second pleasure *B*. In order to maximize the sum of his pleasure, the individual

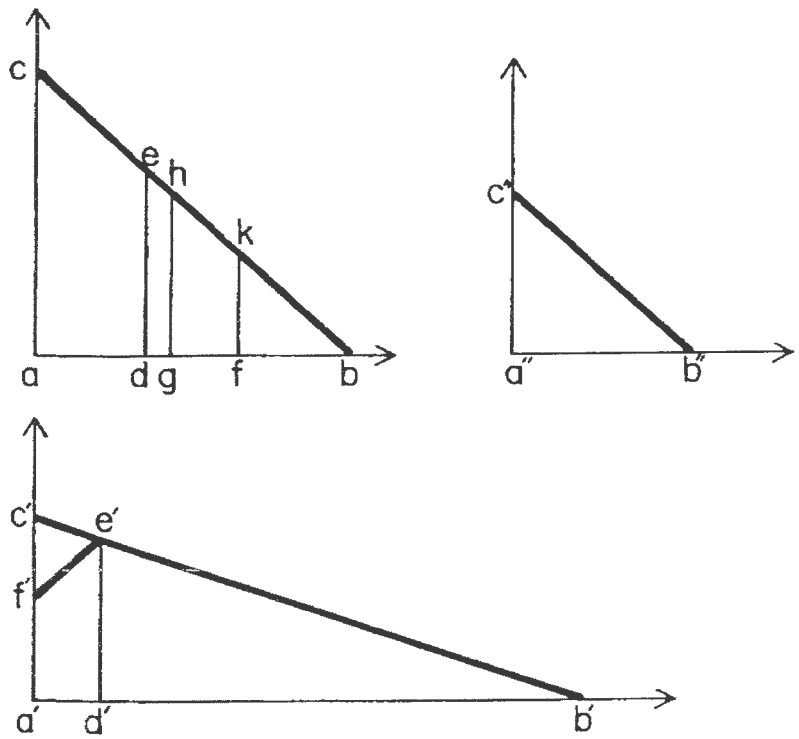

Figure 1.6

must start by satisfying that pleasure that has the greatest intensity at the beginning (in this case, A) and continue its enjoyment until this pleasure [its intensity] sinks so low that it equals the [intensity of] pleasure B at its beginning. That is, he must enjoy A up to d, where $\overline{de} = \overline{a'c'}$. Therefore, should he have only the time $\overline{ad}$ for enjoyment, he must devote it completely to pleasure A. Should he have more time available—say, up to f—and should be choose to devote the entire time $\overline{af}$ to the enjoyment of pleasure A, he would obviously not achieve the maximum of pleasure. Let d' be determined so that $\overline{a'd'} = \overline{fg}$ and $\overline{d'e'} = \overline{gh}$. This can always be done because the lines cb and $c'b'$ approach continuously the horizontals ab and $a'b'$. This condition obtains if $\overline{a'f'} = \overline{fk}$, $\angle a'f'e' = \angle fke$, and $e'd'$ is perpendicular to $a'b'$. Thus the magnitude of enjoyment is measured by $\overline{gfkh}$ when the time $\overline{gf}$ is spent on enjoying pleasure A, and by $\overline{a'd'e'c'}$ when it is allocated to enjoying B. Because of the equality of the sides and of the angles, we have $\overline{gfkh} = \overline{a'd'e'f'}$ and, hence, $\overline{a'd'e'c'} = \overline{gfkh} + \overline{f'e'c'}$. Consequently, if the time is distributed for both pleasures in such a manner that at the termination of enjoyment the same magnitude [intensity] is reached for each pleasure—that is, if $\overline{gh} = \overline{d'e'}$—then the total pleasure is greater by area $\overline{f'e'c'}$ than if pleasure A alone had been

enjoyed during the entire time. It is thus obvious that any other time distribution would result in a reduction of the total pleasure. Only if there is sufficient time for both pleasures to be satisfied completely, can the point b be attained for pleasure A together with the point b' for pleasure B.

The result is not changed substantially if a third, fourth, . . . pleasure is added. Man will always have to utilize the time allowed him for enjoyment by first enjoying the pleasure that has initially the greatest magnitude [intensity] until [the intensity of] this pleasure declines to equality with the next (in figure 1.6, until $\overline{de} = \overline{a'c'}$). From then on, he has to distribute his time between both pleasures A and B, specifically, until $\overline{gh} = \overline{d'e'}$ equals the [initial intensity of the] next largest pleasure—here, $\overline{a''c''}$. Should additional time be available, a threefold distribution will take place and again in such a manner that at the termination of the enjoyment, the marginal segments should be equal to each other, and so forth.

Taking a rectilinear triangle as a basis of discussion, one finds the following formulas for the calculation of the distribution of a given time interval: Let the periods during which pleasures are experienced [to satiety] be $\overline{ab} = p$, $\overline{a'b'} = p'$, and, in general,

$$\overline{a^{(\nu)}b^{(\nu)}} = p^{(\nu)}. \tag{1.1}$$

Let the initial magnitudes [intensities] of the pleasures be $\overline{ac} = n$, $\overline{a'c'} = n'$, and, in general,

$$\overline{a^{(\nu)}c^{(\nu)}} = n^{(\nu)}, \tag{1.2}$$

and let the total available time [the total life of the individual] be E. Finally,[7] let e denote the time spent on pleasure A, e' that on B . . . , and $e^{(\nu)}$ that on N. For the calculation of $e^{(\nu)}$, we have the following equations.

The sum of the $e^{(\nu)}$ must be equal to the total available time; hence

$$E = e + e' + e'' \cdots + e^{(\nu)}. \tag{1.3}$$

Let us also denote the marginal ordinates $\overline{de}$, $\overline{d'e'}$, and, in general, $\overline{d^{(\nu)}e^{(\nu)}}$. Since the marginal ordinates at the points of termination of the enjoyments must all be equal, in each triangle (figure 1.6),

$$\overline{ab} : \overline{ac} = \overline{bd} : \overline{de} = (\overline{ab} - \overline{ad}) : \overline{de}$$

and

$$\overline{de} = \frac{\overline{ac}(\overline{ab} - \overline{ad})}{\overline{ab}} = \frac{\overline{a'c'}(\overline{a'b'} - \overline{a'd'})}{\overline{a'b'}} \cdots .$$

This becomes

$$\frac{n(p-e)}{p} = \frac{n'(p'-e')}{p'} = \cdots = \frac{n^{(v)}p^{(v)} - e^{(v)}}{p^{(v)}}. \tag{1.4}$$

From these equalities, it follows that

$$e' = p' - \frac{p'}{n'}\left[\frac{n(p-e)}{p}\right],$$

and further,

$$e'' = p'' - \frac{p''}{n''}\left[\frac{n(p-e)}{p}\right],$$

and, in general,

$$e^{(v)} = p^{(v)} - \frac{p^{(v)}}{n^{(v)}}\left[\frac{n(p-e)}{p}\right]. \tag{1.5}$$

If we substitute these values in equation (1.3) and factor out $n(p-e)/p$,

$$E = e + p' + p'' + \cdots + p^{(v)} - \frac{n(p-e)}{p}\left(\frac{p'}{n'} + \frac{p''}{n''} + \cdots + \frac{p^{(v)}}{n^{(v)}}\right). \tag{1.6}$$

And if we add $p - (p/n)\,[n(p-e)]$ to both sides and perform the reduction,

$$E = p + p' + p'' + \cdots + p^{(v)} - \frac{n(p-e)}{p}\left(\frac{p}{n} + \frac{p'}{n'} + \frac{p''}{n''} + \cdots + \frac{p^{(v)}}{n^{(v)}}\right). \tag{1.7}$$

Therefore, if we set

$$\frac{p}{n} + \frac{p'}{n'} + \frac{p''}{n''} + \cdots + \frac{p^{(v)}}{n^{(v)}} = \alpha, \tag{1.8}$$

$$p + p' + p'' + \cdots + p^{(v)} = P,$$

(1.7) yields

$$E = P - \frac{\alpha n(p - e)}{p}, \tag{1.9}$$

or

$$e = p\left(1 - \frac{P - E}{\alpha n}\right). \tag{1.10}$$

Similarly [by (1.5)], in general

$$e^{(\nu)} = p^{(\nu)}\left(1 - \frac{P - E}{\alpha n^{(\nu)}}\right). \tag{1.11}$$

With the aid of this value of $e^{(\nu)}$, one obtains the magnitude [intensity] $\overline{de}$ of a pleasure at the point of its termination. Substituting it in the proportion

$$p : n = (p - e) : \overline{de}$$

and reducing yield

$$\overline{de} = \frac{P - E}{\alpha}. \tag{1.12}$$

Finally, in every triangle [such as *abc* in figure 1.6], for the part that measures the amount of the pleasure experienced

$$\text{area } \overline{adec} = \frac{\overline{ac} + \overline{de}}{2} \times \overline{ad} = \frac{n + \overline{de}}{2} \times e$$

$$= \left[\frac{n + (P - E)/\alpha}{2}\right] \times p\left(1 - \frac{P - E}{\alpha n}\right) = \frac{pn}{2} - \frac{p(P - E)^2}{2n\alpha^2}.$$

If we denote by W_1 the total life pleasure that man achieves during the time E by this [optimal] distribution,[8] we obtain [by (1.8)]

$$W_1 = \frac{pn + p'n' + p''n'' + \cdots}{2} - \frac{(P - E)^2}{2\alpha^2}\left(\frac{p}{n} + \frac{p'}{n'} + \frac{p''}{n''} \cdots\right)$$

$$= \frac{1}{2}\left[pn + p'n' + p''n'' + \cdots - \frac{(P - E)^2}{\alpha}\right]. \tag{1.13}$$

It is necessary to mention, however, that these formulas yield correct results only if E is smaller than P or, at most, equal to it. Moreover,

for the calculation of any concrete case, p and n should be taken only for those pleasures that actually take part in the distribution of E. The value of E at the point where the participation of a particular pleasure begins is found by setting the expression of the corresponding e equal to zero in (1.11). In this way, we obtain for every pleasure that is partly satisfied the condition

$$E \geqslant P - \alpha n^{(v)}. \tag{1.14}$$

A numerical example will make these considerations clearer. Let it be assumed for four freely available pleasures that the initial relations of magnitudes [intensities] are in the proportions 10 : 8 : 5 : 2 and that their durations are in the proportions 10 : 16 : 15 : 18. In this case, $n = 10$, $n' = 8$, $n'' = 5$, and $n''' = 2$ and $p = 10$, $p' = 16$, $p'' = 15$, and $p''' = 18$. The pleasure for which $n = 10$ must be enjoyed first. In order to determine how long it should be enjoyed, we substitute $P = p + p' = 26$, $\alpha = (p/n) + (p'/n') = 1 + 2 = 3$, and $n^{(v)} = n' = 8$ in the formula $E = P - \alpha n^{(v)}$. This yields

$$E = 26 - 3 \times 8 = 2.$$

Therefore for only 2 time units, pleasure A alone should be satisfied. If, further, we take $P = p + p' + p'' = 41$, $\alpha = (p/n) + (p'/n') + (p''/n'') = 1 + 2 + 3 = 6$, and $n^{(v)} = n'' = 5$, then

$$E = 41 - 6 \times 5 = 11.$$

And if this value of E is introduced in the formula (1.11) for $e^{(v)}$, it yields $e = 5$ and $e' = 6$.

For less than 11 time units, the distribution of time must therefore be limited to the two pleasures [of greatest initial intensities], and if exactly 11 time units are available, then 5 should be used for the greatest pleasure and 6 for the second greatest pleasure. Finally, if we take $P = p + p' + p'' + p''' = 59$, $\alpha = (p/n) + (p'/n') + (p''/n'') + (p'''/n''') = 1 + 2 + 3 + 9 = 15$, and $n^{(v)} = n''' = 2$, then

$$E = 59 - 15 \times 2 = 29.$$

In this case, from (1.11) we obtain $e = 8$, $e' = 12$, and $e'' = 9$.

Of the 29 time units, 8 will have to be used for the greatest, 12 for the second-greatest, and 9 for the third-greatest pleasures. If additional time

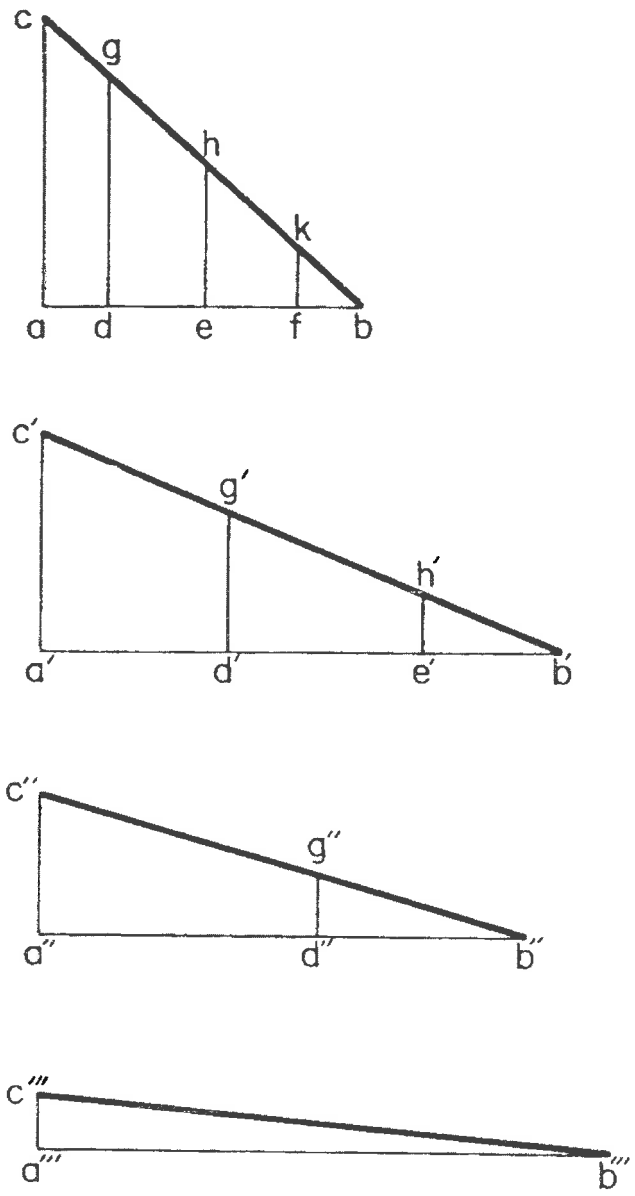

Figure 1.7

became available, then also the fourth pleasure would be partially satisfied until for 59 time units, all pleasures would be satisfied to satiety.

Figure 1.7 illustrates these points: abc represents the diagram of one pleasure for which $p = \overline{ab}$ and $n = \overline{ac}$, both being equal to 10; $a'b'c'$ represents the case of $p' = \overline{a'b'} = 16$ and $n' = \overline{a'c'} = 8$; $a''b''c''$ represents the case of $p'' = \overline{a''b''} = 15$ and $n'' = \overline{a''c''} = 5$; and finally, $a'''b'''c'''$ represents the case of $p''' = \overline{a'''b'''} = 18$ and $n''' = \overline{a'''c'''} = 2$. If only the time $\overline{ad}$ is available, enjoyment must be limited to pleasure A and $\overline{dg} = \overline{a'c'}$; if the time available equals $\overline{ae} + \overline{a'd'}$, then pleasure A will be satisfied up to point e, and pleasure B up to point d', because $\overline{he} = \overline{d'g'} = \overline{a''c''}$. If the [available] time equals $\overline{af} + \overline{a'e'} + \overline{a''d''}$, then, with the equalities $\overline{fk} = \overline{e'h'}$, and $\overline{d''g''} = \overline{a'''c'''}$, A is satisfied up to f, B up to e', and C up to d''. Should there be even more time available, then also pleasure D will finally be satisfied partially. Table 1.1 may show this more clearly.

From this table it is seen that after B enters into the picture, the [additional] time must be divided between A and B in the proportion 1 : 2, which means that it must be allocated $\frac{1}{3}$ to A and $\frac{2}{3}$ to B. After C is brought in, A receives only $\frac{1}{6}$, B receives $\frac{1}{3}$, and C receives $\frac{1}{2}$, and, finally, with the entry of D, A receives $\frac{1}{15}$, B receives $\frac{2}{15}$, C receives $\frac{1}{5}$, and D receives $\frac{3}{5}$ of the additional time.

A diagram of the total pleasure generated by this procedure can be obtained with the aid of formula (1.12) for $\overline{de}$. We shall premise that the time at the disposal of the individual for the satisfaction of the various pleasures increases gradually, beginning at zero. No matter what we assume about the length of this time period, we shall always find, in accord with the foregoing argument, that the magnitudes [intensities] of all pleasures at their moments of termination are equal to $(P - E)/\alpha$. If one erects the perpendiculars on the line ab proportional to the magnitudes [intensities] corresponding to this formula, the area above ab must represent the total life pleasure, or W_1. This can be also shown directly by differentiating W_1 when W_1 and E are treated as variables. It is clear that if W_1 represents the total life pleasure, the differential coefficient dW_1/dE represents the magnitude of pleasure added to W_1 during the time dE. The differentation of (1.13) with respect to E yields

$$\frac{dW_1}{dE} = \frac{P - E}{\alpha}. \tag{1.15}$$

And if w_1 denotes this magnitude,[9] one obtains the equation of the curve

Table 1.1

E	e	e'	e''	e'''
2	[2]	[0]		
3	$2\frac{1}{3}$	$\frac{2}{3}$		
4	$2\frac{2}{3}$	$1\frac{1}{3}$		
5	3	2		
6	$3\frac{1}{3}$	$2\frac{2}{3}$		
7	$3\frac{2}{3}$	$3\frac{1}{3}$		
8	4	4		
9	$4\frac{1}{3}$	$4\frac{2}{3}$		
10	$4\frac{2}{3}$	$5\frac{1}{3}$		
11	5	6	0	
12	$5\frac{1}{6}$	$6\frac{1}{3}$	$\frac{1}{2}$	
13	$5\frac{1}{3}$	$6\frac{2}{3}$	1	
14	$5\frac{1}{2}$	7	$1\frac{1}{2}$	
⋮	⋮	⋮	⋮	
28	$7\frac{5}{6}$	$11\frac{2}{3}$	$8\frac{1}{2}$	
29	8	12	9	0
30	$8\frac{1}{15}$	$12\frac{2}{15}$	$9\frac{1}{5}$	$\frac{3}{5}$
31	$8\frac{2}{15}$	$12\frac{4}{15}$	$9\frac{2}{5}$	$1\frac{1}{5}$
32	$8\frac{1}{5}$	$12\frac{2}{5}$	$9\frac{3}{5}$	$1\frac{4}{5}$
⋮	⋮	⋮	⋮	⋮
44	9	14	12	9
⋮	⋮	⋮	⋮	⋮
59	10	16	15	18

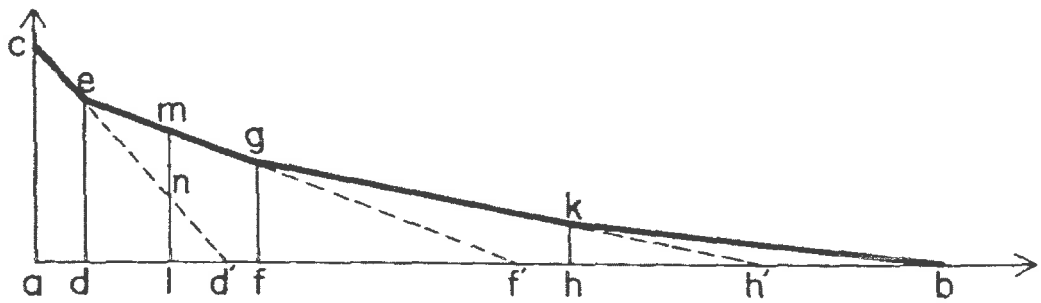

Figure 1.8

in question [that is, of the curve *cemgkb* of figure 1.8]:

$$w_1 = \frac{P - E}{\alpha}. \tag{1.16}$$

In this connection, it should be observed that α and P change every time E reaches a value such that a previously unattained pleasure can now be satisfied partially.

For our numerical example, we have the following results:

1. Until E reaches 2, $\alpha = p/n$ and $P = p$; hence $w_1 = (10 - E)/1$.
2. From $E = 2$ until $E = 11$, $\alpha = (p/n) + (p'/n')$ and $P = p + p'$; hence $w_1 = (26 - E)/3$.
3. From $E = 11$ until $E = 29$, $\alpha = (p/n) + (p'/n') + (p''/n'')$ and $P = p + p' + p''$; hence $w_1 = (41 - E)/6$.
4. From $E = 29$ until $E = 59$, $\alpha = (p/n) + (p'/n') + (p''/n'') + (p'''/n''')$ and $P = p + p + p' + p'' + p'''$; hence $w_1 = (59 - E)/15$.

The values of W_1 and w_1 as functions of E lead to table 1.2.

The geometric representation of figure 1.7 is given at a scale reduced by half in figure 1.8. Since we have taken here $\overline{ac} = 10$ for the case of $E = 2$, it follows that $\overline{ad} = 2$, $\overline{de} = 8$ and area $\overline{adec} = [(10 + 8)/2] \times 2 = 18 = W_1$; further, $\overline{af} = 11$ and, hence, $\overline{af} - \overline{ad} = 9$, $\overline{fg} = 5$, and area $\overline{dfge} = [(8 + 5)/2] \times 9 = 58.5$, which yields area $\overline{afgec} = 18 + 58.5 = 76.5 = W_1$ for $E = 11$, and so on. (From the formula $w_1 = (P - E)/\alpha$ it follows that $\overline{ce}$, $\overline{eg}$, $\overline{gk}$, and $\overline{kb}$ are straight lines and that, in general, the limiting line [representing w_1] is a straight line so long as α and P maintain the same values, in other words, so long as the distribution [of time] is to be made among the same set of pleasures. That *ce*, *eg*, ... are straight lines results from the use of a rectilinear triangle in the diagram of pleasure.) As a consequence of this proposition, the diagram of the total life pleasure can be represented in general by a curve cb' that approaches *ab* continuously and is convex toward this line (figure 1.9).

From all this, we can see that with the increase of the time of enjoyment, the total life pleasure W_1 grows continuously up to the point when the time permits the satisfaction of all pleasures to satiety. Formula (1.13) for W_1 shows immediately that this proposition must be true under all circumstances. The reason is that with the increase of E, the subtrahend

Table 1.2

E	w_1	W_1
0	10	0
1	9	9.5
2	8	18
3	$7\frac{2}{3}$	25.833
4	$7\frac{1}{3}$	33.333
5	7	40.5
6	$6\frac{2}{3}$	47.333
7	$6\frac{1}{3}$	53.833
8	6	60
9	$5\frac{2}{3}$	65.833
10	$5\frac{1}{3}$	71.333
11	5	76.5
12	$4\frac{5}{6}$	81.417
13	$4\frac{2}{3}$	86.167
14	$4\frac{1}{2}$	90.75
⋮	⋮	⋮
28	$2\frac{1}{6}$	137.417
29	2	139.5
30	$1\frac{14}{15}$	141.467
31	$1\frac{13}{15}$	143.367
⋮	⋮	⋮
58	$\frac{1}{15}$	169.467
59	0	169.5

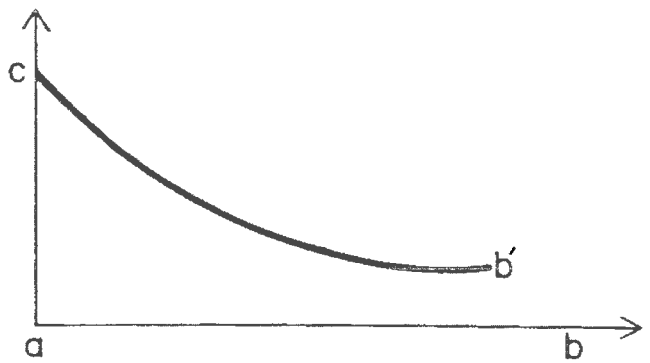

Figure 1.9

$(P - E)^2/\alpha$ becomes continuously smaller, so that W_1 increases continuously. But even though W_1 becomes larger with the extension of the time for enjoyment, this increase is by no means proportional to this extension of the available time. In our numerical example, the first 29 time units—which amount to less than half of the time sufficient for all pleasures to become completely satisfied—yield a life pleasure equal to 139.5. By contrast, the last 30 time units—representing more than half of the total time—yield only 30 units of pleasure, that is, approximately $\frac{2}{9}$ of those of the first 29 time units. The difference is even larger if we compare the magnitudes of pleasure satisfied during each time unit. The first unit produces a life pleasure equal to 9.5; the last unit, a life pleasure equal to $169.5 - 169.467 = 0.033 = \frac{1}{30}$, that is, merely $\frac{1}{285}$ of the pleasure that the first unit gives. The formula $w_1 = (P - E)/\alpha$, which represents the increment of life pleasure, shows that this relation is valid generally: w_1 becomes smaller as E becomes larger. A direct consequence of this proposition is the following:

[1.3] *Man has the possibility of further increasing his total life pleasure when prevailing circumstances permit him to discover a new pleasure, however small by itself, or to increase an already known pleasure by developing himself or, alternatively, affecting the external world.*

The increase of a known pleasure can only occur if its absolute magnitude [intensity] is increased at every moment or, at least, at some moments.[10] This means that the line $\overline{cb}$ (figure 1.10)—in its entire length or, at least, in part—moves further away from $\overline{ab}$ than it was previously. (The case in which the increase is obtained by a possible extension of the time of enjoyment is, in essence, not different. In this case the increase occurs for those moments at which the [intensity of] pleasure was previ-

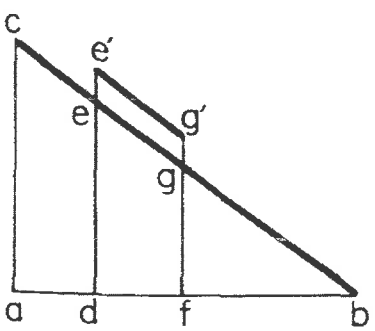

Figure 1.10

ously zero; the same is true for the discovery of a new pleasure.) Consequently, if area $\overline{defg}$ represents the magnitude of pleasure during the time interval $\overline{df}$ and if it has been possible to achieve an increase in the intensity of pleasure during this interval, this change must be represented by the increase of the segments $\overline{de}$ and $\overline{fg}$, as is illustrated by $\overline{de'}$ and $\overline{fg'}$. Should the pleasure in fact be satisfied during the time $\overline{df}$ according to the above proposition, the total pleasure will be increased by area $\overline{egg'e'}$, even if no consideration at all is given to a redistribution of time, so that the time distribution remains unchanged. But with this [unchanged] distribution, the limit line of the pleasure under discussion would be higher than those of the other pleasures satisfied up to this point. Hence the sum of all pleasures can be increased further by an appropriate redistribution of the available time. And if not all pleasures are satisfied to satiety, such redistribution of time will increase the total pleasure by more than the increase of that particular pleasure taken by itself.

The formula for w_1 leads to the same conclusion, namely: The total life pleasure must increase if w_1 becomes larger *at every single moment in time* [italics added] and must decrease if the contrary is the case. The formula (1.16),

$$w_1 = \frac{P - E}{\alpha},$$

shows that the larger is P, the more time will transpire before w_1 becomes zero. That is, the decrease in the value of w_1 proceeds more slowly and w_1 remains at high values for a longer time span than in the previous situation [for the old value of P]. But the magnitude of P, being the time necessary to enjoy all pleasures to satiety, is independent of the absolute magnitudes of the pleasures.[11] This time it increases due to the discovery of a new pleasure or the prolongation of enjoyment [to satiety] of a known pleasure. Figure 1.8 can show this clearly if the lines ce, eg, and gk are extended until they intersect ab. The extension of ce intersects ab already at d'. With only the first pleasure, the enjoyment would end at point d'. With the entry of the second pleasure, the [end] point is moved farther out to f'; as a result, the [intensity of] pleasure from point d on remains greater. For example, at l it is now $\overline{lm}$ instead of $\overline{ln}$, and similarly at every other point.[12] The same is true for the case of a newly discovered pleasure.

Formula (1.6) shows, further, that w_1 becomes larger as α becomes smaller; but the latter, being equal to $(p/n) + (p'/n') + (p''/n'') + \cdots$ [by

(1.8)], will become smaller whenever one of the n becomes larger, that is, if the absolute magnitude of a known pleasure increases.[11]

The discovery of a new pleasure or the increase of a known pleasure could be a matter of indifference to man only if he were to reach the stage of having, during the entire duration of his life, the choice of so many and such great pleasures that a newly discovered pleasure or increased pleasure does not come into play. How very far man is from such a situation is again shown most clearly by the case of Louis XV, who had the advantage over most other men of possessing means in abundance for procuring all known pleasures in any desired measure. He would not have fallen into the mistake of misusing the means at his disposal for the purpose of overindulging himself had he known how to occupy his time always with new and real pleasures.

As long as a continued decrease of the magnitude [intensity] of pleasure takes place, a modification of the law of decrease cannot change the above propositions [concerning w_1]. It can only cause the repetition of a pleasure to come about more or less frequently or a little earlier or later and to be distributed differently over time. In general, such modifications do not alter the fact that there must be a limitation of repetitions, and if time does not suffice for achieving satiety of all pleasures, then before satisfying the greatest pleasure completely, even the smallest must be enjoyed and all must be satisfied up to the point of equal magnitude [intensity] when terminated.

From this there results the following rule for human conduct: These laws can help man to achieve the purpose of his life to the extent permitted by Creation only if all the effective forces of Creation are completely known to him; for only then will man be certain of knowing all the pleasures available to him and the means for their increase. Hence the first rule for his conduct is,

Bend all effort to obtain this knowledge.

We have already observed how the Creator arranged the laws of enjoyment in such a way as to achieve the absolute certainty that the human race will not cease to progress in art and science until it has reached the goal that the Creator has set for it in His unfathomable wisdom.

With the very first steps toward this knowledge, however, man realizes that the comprehension of the laws of Creation does not suffice for accomplishing his life purpose; rather, for the accomplishment of true

enjoyment, he must—once in possession of this knowledge—act upon the external world with the intent of giving it that form that alone can produce the desired effects. That is not all; it also becomes manifest that man's powers are by no means sufficient to bring about immediately any desired change in the external world and that, moreover, his isolated effort can produce only a very insignificant change in the external world. But it also becomes apparent that the manner of this action has a highly significant effect in that it may be more or less advantageous for man's objective in life. Furthermore, the magnitude of this influence will grow in far greater proportion than the increase of physical power when by unified effort men strive to achieve the same end. It thus becomes of paramount importance to answer the following question: *How can we ascertain whether through an individual or collective action the form of the external world has been improved in regard to man's life purpose?* All investigations, then, should be directed toward finding a yardstick by which the different situations of the external world may be compared.

We shall express the condition of the external world that makes it helpful in reaching the realization of our life purpose as follows:

[A.3] *The external world has value for us,* from which it follows *that the value of the external world for us increases or decreases in direct proportion to the help it gives us in attaining our life's purpose and that, consequently,* **the magnitude of its value is measured exactly by the magnitude of life pleasure that it gives us.**

If we now regard the external world from this viewpoint and try to arrive at an estimate of its worth, it is appropriate to classify all external objects into three categories.

First are objects provided by nature or produced by human labor in such a way that they possess all the properties necessary to provide some specific pleasure; hence all that is needed for their actual enjoyment is to bring them into contact with our body organs. We shall refer to such objects as *means of enjoyment.*[13]

The apple grown in the wild and eaten raw has been endowed by nature with all the qualities appropriate for satisfying a specific pleasure; the cook has given to his dish, the tailor to his coat, the cabinetmaker to his chair, the builder to the house, the painter to the painting, and so forth, all the properties that render these objects capable of satisfying directly the intended pleasures. Their value will therefore be measured exactly by the

magnitude of pleasure that man actually obtains through them. Therefore it makes a great deal of difference in regard to the value of these objects whether an object of this type is consumed completely in one single act of enjoyment or permits several repetitions of one and the same pleasure. Unlike the wild apple or the dish of food, which has only that value corresponding to the magnitude of pleasure obtained through one single act of alleviating hunger, the value of the coat or the chair is equal to the sum of all the single pleasures that I experience as I make use of this object until it is completely worn out. For an individual, therefore, the value of a house or a painting—each an object that ordinarily lasts longer than a human life time—is equal to the sum of all single pleasures that he can derive from any one of these objects during his entire life. In the propositions on enjoyment developed earlier, we have seen, however, that for each given pleasure a maximum is reached—varying with the education of each individual—as the same pleasure is repeated with a certain regularity in relation to the flow of time. It follows that the total pleasure increases for each individual in relation to time and that the value of those objects that can be used repeatedly and with the same effect for the satisfaction of pleasure—or, as we say, that do not wear out from repeated use—usually increases exactly in proportion to the time during which they yield service. For other objects, which undergo deterioration through time, this increase is small in proportion to the decrease in the magnitude of pleasure because of their deterioration, or, rather, the time during which their services last becomes shorter [with greater frequency of use]. To this class belong, among other things, the surface of the earth insofar as it is used for the construction of dwellings and, in general, the land and all nature insofar as they offer enjoyment only through their contemplation and their potential for amusement. To the same category belong the gardens that serve solely for this purpose, as well as human labor insofar as it is employed for personal services and entertainment. The functions of, for example, a companion or of a servant generally belong to this class.

Second, in contrast, are those objects—useful to us in the satisfaction of pleasure and, therefore, of value—for which the inseparable union of all the qualities necessary to satisfy the intended pleasures is either impossible or, at the least, not yet realized. In order that a furnace may provide us the intended pleasure, namely, heat, it needs fuel and fire; a wagon needs some motive power and a driver; a pipe needs tobacco and light; an organ, a violin, a flute needs a musician who knows how to play this

instrument; and so on. It is impossible to produce any of these objects in such a fashion that the required complement can be left off without disadvantage. Rye and wheat are intended for the baking of bread, cloth for clothing, paint and canvas for the making of a painting, but all have to await the transformation by labor in order to become means of enjoyment. In the case of all these goods, an evaluation is possible only when they present themselves in a union [with other goods or services] in such a way that they can actually provide pleasure. And in these combinations, the aggregate value is equal to the magnitude of pleasure that can be obtained from them. It is impossible, however, to determine precisely how this value should be distributed among the individual parts, which generate pleasure through their joint action, precisely because they have value only when they are present in some definite combination. Apart from this combination, they have value only insofar as there is a prospect that the requisite components can be obtained to complete what is missing. For this very reason, the evaluation of the individual parts is very different under different circumstances. Depending on whether it is more or less difficult for a person to obtain one or the other of the component parts, the valuation of the other components will rise or fall in such a way that the sum of the value of all individual parts will reach exactly the level that corresponds to the magnitude of the pleasure satisfied. The oven, together with the fuel and the fire, has a value equal to the pleasure of the warmth generated by it; the specific quantity of rye, including the labor of the baker, equals in value the pleasure of the bread produced from it. Unfortunately, however, the distribution of this value between the furnace, the fuel, and the fire, or between the rye and the labor of the baker, does not allow closer determination, but depends on the prevailing circumstances. If a person has at his disposal, for example, a furnace and some fuel, then the fire gets the entire value that heating has for him, just as, alternatively, the furnace or the fuel will obtain this value if he possesses either the latter or the former as well as the fire. The only thing to be remembered here is that the sum of the value of the components generating pleasure through joint action is equal to the magnitude of the pleasure itself.

Linguistic usage denotes objects that belong to this class by very diverse terms, such as tools, luxury goods, instruments, materials [in general], raw materials, and manufactured goods—terms that neither are limited only to objects of this second category nor embrace all goods belonging to it. In what follows, I shall refer to them as *objects of the second category*.

The third and last category of objects that have value, and that therefore I shall denote as *objects of the third category*, are those that are useful in the production of means of enjoyment and their parts, but that never themselves become means of enjoyment or parts of these means. Land insofar as it serves to produce commodities belongs to this category, and so does the oil, the fuller's earth used by the cloth maker, the fuel for heating machinery, and, in general, everything that the craftsman, the producer, and the artist use up as materials, but that cannot subsequently be identified in the means of enjoyment. To this category also belong all tools and machines that man has invented in order to strengthen and perfect his mechanical power and, finally, the work of domesticated animals and, above all, human labor itself. For all these objects, the evaluation can only be indirect, and value can be ascribed to them only insofar as they aid in the production of some means of enjoyment or of a component of it; hence to these commodities applies all the more what has been said about the way in which the evaluation is to be performed in the previous category. They have value corresponding exactly to their contribution in the production of means of enjoyment, and hence the proposition established above for the goods of the second category holds here too, namely: We can determine only the sum of the value that all these objects have jointly from serving in the production of some means of enjoyment. The determination, however, of the fractions of total value that falls to each part depends on the special circumstances, and furthermore, the value of the last component that would be required for the satisfaction of pleasure constitutes exactly the difference from the sum representing the magnitude of pleasure. Thus each component can obtain a value equal to the sum in case everything else is available [free] to man for partaking in a specific pleasure except that particular necessary component.

Apart from this, the value of these objects, even if only indirectly, can nevertheless reach unusually high levels. The reason is that most of these objects can serve, more or less often, in the production of means of enjoyment and their integral parts—many of them, if properly handled, even with increasing advantage, as is the case of human labor or land used in agriculture. The value attributable to them for such service in the process of producing means of enjoyment or parts thereof is here multiplied as often as the repetition of the service; hence the value of such an object is equal in this case to the result of this multiplication.

For the objects of the second and third categories, we should recall the

observation pertaining to the means of enjoyment that may serve repeatedly in satisfying pleasure. In general, this service too can be repeated only sequentially, not simultaneously. The value of such objects increases, therefore, generally in proportion to the period of service.

In addition to the objects of these three categories, there are also those that have value because they help in the production of objects of the third category, and still others that help produce the latter—a sequence that may be continued conceivably *ad infinitum*. These objects, however, do not constitute a separate category; they belong to the third category because the principles according to which they are to be appraised are exactly the same as in the case of the objects of the third category.

A further observation should be made here. The Creator built for man a world extraordinarily rich in pleasures, so that, in general, an object can serve to satisfy pleasure in a variety of ways; frequently, the same object can even produce different pleasures either simultaneously or sequentially. Wheat can be used as nourishment or in the production of starch; similarly, oil can be used as food, for obtaining light, or for lubricating our man-made machines;[14] and so on. A fruit tree gives us pleasure for many years through its foliage and its blossoms; it refreshes us with its shade and its fruits; and in the end it gives us its wood, a precious material for furniture or heating. Man has taken advantage of the capacity of objects to produce various pleasures either simultaneously or sequentially, and so we see the cook and the pastry maker striving not only to prepare tasty dishes but also to give them shape and color pleasing to the eye. For the same reason, the cabinetmaker aims not only to make the chair just comfortable to sit on but also to upholster it with fabrics agreeable to the eye, to polish it, and so forth, so that it is pleasing both to the eye and touch. The unlimited wealth, however, of the pleasure-generating powers in nature does not require an essential modification of the laws according to which valuation is to be made. When, as with wheat and oil, a choice must be made between the pleasures to be satisfied by their use, the value is determined in accordance with the above principles once this choice has been made. In those cases where there is an accumulation of pleasures, the value is to be determined for each individual pleasure according to the above principles, and the value of the object is equal to the sum of the values determined in this manner. (The still existing difficulties with the practical execution of this determination of value will be overcome later.)

If we examine more closely the manner in which the satisfaction of

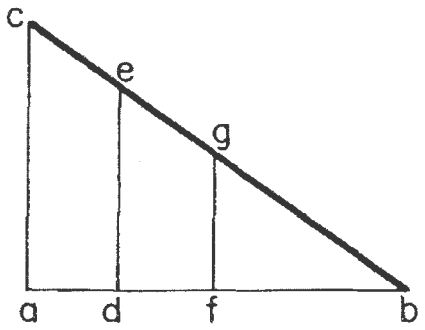

Figure 1.11

pleasures by the external world takes place, we find that for nonmaterial pleasures the diagram of enjoyment also serves directly as the diagram of value of the means of enjoyment *because in their case the yardstick of the [value of] possession is the duration of enjoyment*. The representation thus obtained does not apply with less justification to material pleasures. For a proof, we shall first examine the means of enjoyment that are consumed through one single use. In their case, the quantity used increases in proportion to the time during which they provide us with our pleasure. And this is not determined just by direct observation; it also follows from the known laws of the working of the forces of nature. In order to obtain a presentation of the value of the means of enjoyment, one needs only to represent by $\overline{ab}$ (figure 1.11) the quantity of the means of enjoyment required to satisfy the pleasure during the period $\overline{ab}$. This representation is valid because in this case, the quantity consumed is proportional to time. It is then obvious—because value is measured by the magnitude [intensity] of pleasure—that the value of each atom of the quantity $\overline{ab}$ is equal to the line perpendicular to the line *ab* at the corresponding point. For example, the value of the atom [represented by the point] *d* equals $\overline{de}$, and the value of any part of the quantity, such as $\overline{df}$, equals [the area of] the trapezoid $\overline{dfge}$ erected on *df*. Also, the value of the entire quantity $\overline{ab}$ is equal to [the area of] the triangle $\overline{abc}$. Thus if in the process of satisfying pleasure point *b*—the instant of temporary satiety—is reached, at that moment a greater quantity has no value at all. It acquires value again at that instant when the repetition of the pleasure appears reasonable; at that moment, the situation described repeats itself. That is, the quantity $\overline{ab}$ acquires value again—specifically, the atom *a* acquires the value $\overline{ac}$, and each subsequent atom gradually acquires a lower value until at *b* zero is reached once more, and similarly with each subsequent repetition.

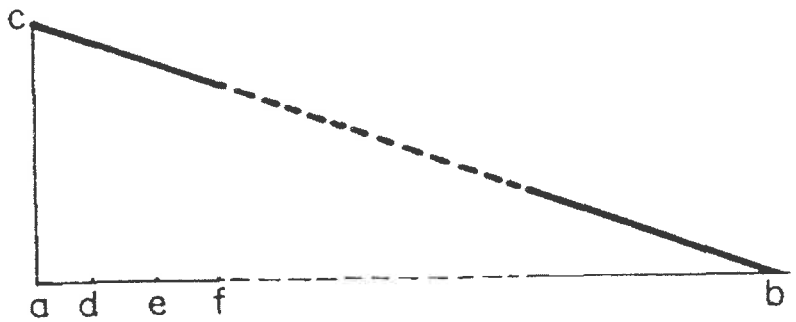

Figure 1.12

We now arrive at a suitable synthesis of the various presentations of the value of both nonmaterial and material means of enjoyment that comes about with each repetition of enjoyment. Let us think that all the first atoms of time or mass that were used in each repetition for the satisfaction of pleasure are arrayed on the line *ab* (figure 1.12), starting with *a* and ending at *d*; similarly, from *d* on, all second atoms are so arrayed as to extend to *e*, so that $\overline{ad} = \overline{de}$; next, all the third atoms are lined up on $\overline{ef}$; and so on. If one proceeds in this way to the last atoms that still yield pleasure, the line segment $\overline{ab}$ must be equal to the sum of all base lines that represent the quantity [necessary] for each repetition of pleasure. If one now draws $\overline{ac}$ equal to the magnitude [intensity] of pleasure at the beginning of each enjoyment and if one connects *c* with *b*, then the [area of the] triangle $\overline{abc}$ gives the sum of the values that the means of enjoyment yield in totality during all repetitions. Here, again, the perpendicular at any point on the line *ab* shows the value of the corresponding atom of the means of enjoyment; also, the [area of] the trapezoid erected on any portion of *ab* represents the value of this portion.

The representation thus obtained of the value of a means of enjoyment is based on the assumption that it continues to maintain, for the entire period during which it is used for the satisfaction of pleasure, the very same characteristics that qualified it as a means of enjoyment for its first use. There actually is no means of enjoyment that fulfills this assumption, for all are subject to a more or less rapid change. Occasionally, the change brings about an improvement, but as a rule it causes a deterioration. The value of the means of enjoyment undergoes an increase or a decrease depending on this change. Changes in the qualities of the means of enjoyment are not the only ones to cause a change in its value; changes originating in man himself also have an equally great influence on the magnitude

of value. Finally, at any given moment, the probability of a repetition of a pleasure has an influence on the determination of the value of its means of enjoyment. The reason is that the value of an expected repetition of a pleasure in the future decreases proportionally with the increase in the probability that for one reason or another the pleasure will not be satisfied. All these factors together imply that the differences between the values of the single atoms of the means of enjoyment are accentuated. It is possible, however, to take into consideration these changes in the representation of value if we change the perpendiculars representing the magnitudes [intensities] of values in the same proportion. All these changes of value have in common that they increase with time, but not in proportion to it. In the reordering of the atoms, therefore, these modifications affect the shape of the diagram in only one respect—the relation of the height of the triangle to its base line will be different. The greater is the value of an atom, the closer it is placed to *a*. Hence the general law remains valid:

[1.4] *The single atoms of one and the same means of enjoyment have very different values, and, in general, for each individual only a definite number of atoms, that is, a definite quantity, has value. An increase in this quantity beyond this point is without any value for that individual, but this point of no value is reached only after the value has little by little moved through many gradations of magnitude.*

If we consider the case of a means of enjoyment when the quantity of its atoms in the possession of an individual is steadily increased little by little, it follows that

[1.5] *With the increase in that quantity, the value of each additional atom must decrease steadily until it sinks to zero.*

Since with the increase in quantity this decrease in value occurs even in the case of means of enjoyment—which are used up in the process of enjoyment—for the means of enjoyment that can be used repeatedly, such loss in value is naturally accelerated in proportion to the possibility of repeated use for the satisfaction of pleasure. For in that case, the possibility of repeated use has precisely this result: If repetition takes place, the quantity used up is correspondingly smaller; therefore man can reach his complete satiety with a proportionately smaller quantity of these means of enjoyment. Were a chair completely consumed in one use, man would

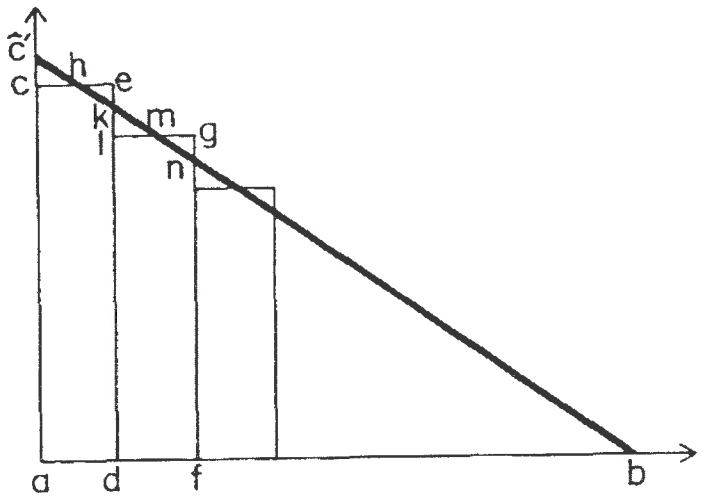

Figure 1.13

need as many chairs as the number of times he wanted to procure for himself the pleasure of sitting down. But because a chair can be used repeatedly, a man needs fewer chairs, the more often a chair can be used. Moreover, all the propositions established earlier are applicable here subject to one single modification. Because the chair cannot be decomposed into atoms in the same manner as the consumable means of enjoyment without losing the property that makes it yield pleasure, we can no longer regard every point on the line *ab* [in figure 1.12] as representing an atom of the means of enjoyment and assign to it a definite value. In this instance, atoms no longer have value by and in themselves but only grouped together to make up the chair—as in our example. Only a similar grouping of points of the line *ab*, a portion of it such as $\overline{ad}$ (figure 1.13), can be used here for the representation of the means of enjoyment. As long as it is used for this representation, this segment must be regarded just as indivisible as the atoms constituting the chair. The value that these atoms have jointly can then be represented by [the area of] a rectangle, such as $\overline{adec}$, erected on $\overline{ad}$. To represent the value of a second chair, we obviously need a segment $\overline{df} = \overline{ad}$ on the line *ab*; hence [the area of] the rectangle that represents the value of the second chair—in this case $\overline{dfgl}$—must be the smaller, the smaller is its value. In this instance, the area representing the value is thus limited by the stepped line *celgn*. We may nevertheless retain the previous form without prejudicing its validity if we draw *c'b* so that [the area of] triangle $\overline{chc'}$ = [the area of] triangle $\overline{hek}$, [the area of] triangle $\overline{klm}$ = [the area of] triangle $\overline{mgn}$, and so on, provided we keep

in mind that the line *ab* can now no longer be cut in any desired segments but only in segments equal to $\overline{ad}$ or some multiple of $\overline{ad}$.

The objects of the second category have value only to the extent that they can serve in some definite combination as means of enjoyment; hence everything that has been said regarding the determination of value of means of enjoyment has direct applicability to this group as a whole. This is no less true for each integral part taken separately since the proportion in which this part participates in the satisfaction of pleasure always remains the same. Otherwise, the premise of the equality of pleasure, which must be maintained in all cases, would be negated. Consequently, the value of such a part must increase or decrease according to the same laws determining the value of the total effect.

The same is also true for the objects of the third category. They have value only insofar as they are useful in the production of means of enjoyments or their parts. Here too, the proportion according to which they render assistance, whatever this proportion may be, always remains the same. Their value must therefore increase or decrease according to the same law as the value of the product since the creation of this product is the only reason for attributing value to these objects.

We find, therefore, that law [1.4] about the decrease of value caused by the increase in quantity has universal applicability for all things that have value. In its most general form, this could be stated as follows:

[1.6] *For all things that may posses value, only a certain quantity—which may be greater or smaller—has value. An increase in the quantity beyond this point is worthless. With an increase in quantity, a commodity steadily approaches this point of no value; thus the first unit of something possessing value has the highest value, and each addition of an equal magnitude has a smaller value until, finally, worthlessness is reached.*

Regardless whether the triangle depicted be curvilinear or rectilinear, it offers a general geometric representation of value. The basis for this statement is that in enjoyment there is present the same law of diminution [of pleasure].

For human conduct, it follows from this law that for the fullest accomplishment of his purpose in life, man in providing himself with means of enjoyment has to abide by rules similar to those found for the distribution of time when this time does not suffice for all pleasures to be satisfied to satiety (see the statement of law [1.2] and accompanying text).

[1.7] *If the individual's powers are insufficient for providing himself fully with all possible means of enjoyment, he must then provide himself with each means of enjoyment up to the point where the last atom of every means shall represent an equal value to him.*

The proof given earlier and the formulas established there apply here without qualification since we have seen that the graphical representation of value is identical to that of pleasure, so that the maximum value is determined under exactly the same conditions as the maximum pleasure. This proposition is obviously nothing else than the transposition to the means of enjoyment of what has been discovered for enjoyment. This transposition is admissible since during a given time interval, the quantity of the means of enjoyment consumed for the satisfaction of a particular pleasure grows with time regardless of any change in the magnitude of pleasure itself during that interval.

The practical application of this rule presents no difficulty once we have succeeded in ascertaining the magnitude of value for each atom of a means of enjoyment or for a particular set of atoms, a point to which I shall return later. Man should leave a pleasure unsatisfied only at that moment when the value of the corresponding means of enjoyment sinks below the determined minimum. This proposition assumes, however, an incomparably greater importance here than in the earlier case because in our circumstances the time at the disposal of an individual is, with rare exceptions, always sufficient for him to satisfy to satiety all possible pleasures as often as it appears reasonable. By contrast, even with the exertion of all his powers, the individual can succeed only in a limited measure in procuring the means of enjoyment.

As is well-known, the last difficulty has brought into existence a special science: political economy. This science sets for itself the task of developing the rules governing the provisioning of the human race with the so-called material goods and how the most advantageous results of this process can be achieved. It thus limits the applicability of its rules to the so-called material goods. There is absolutely no good reason for this limitation since man engaged in enjoyment is completely indifferent whether the pleasure is created through material or nonmaterial goods. This limitation was imposed solely by the circumstance that it seemed impossible to formulate rules applicable above and beyond the material goods. The present conventional name of this science is no longer appro-

priate if we set aside this limitation and extend the purpose of this science to its real dimensions—*to help man obtain the greatest sum of pleasure during his life*. With this idea in mind, in the sequel I shall speak instead of the *science of pleasure*. In the proposition just enunciated, we have discovered the most universal law, for we should not forget that not only material goods but also nonmaterial things, insofar as they render pleasure, belong to the category of means of enjoyment and that for them the period of enjoyment yields a measure of possession [value].

This proposition expresses the rule of conduct for obtaining the maximum of life pleasure only in regard to the supply of the objects of the first and second categories. Indirectly, however, it applies also to the objects of the third category. For these last objects have value only because and to the extent that they serve in the production of objects of the first two classes. From this it follows *that their procurement is to be carried out to a degree that appears desirable when the means of enjoyment are produced in quantities that are reasonable according to what has been said.*