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Review

The Acquisition of Modal Concepts

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Sometimes we accept propositions, sometimes we reject them, and sometimes we take propositions to be worth considering but not yet established, as merely possible. The result is a complex representation with logical structure. Is the ability to mark propositions as merely possible part of our innate representational toolbox or does it await development, perhaps relying on language acquisition? Several lines of inquiry show that preverbal infants manage possibilities in complex ways, while others find that preschoolers manage possibilities poorly. Here, we discuss how this apparent conflict can be resolved by distinguishing modal representations of possibility, which mark possibility symbolically, from minimal representations of possibility, which do not encode any modal status and need not have a logical structure.

Highlights

Infants and non-human animals pass tasks that show that they can run simulations and generate hypotheses. Do infants and non-human animals have modal concepts?

Children struggle with simple tasks that require them to consider multiple possibilities simultaneously until about age 4. This is surprising if infants have modal concepts.

A distinction between minimal representations of possibilities and modal representations of possibilities resolves this tension. Only modal representations of possibilities use modal concepts. Minimal representations of possibilities are sufficient for passing the infant and non-human animal tasks.

Children start talking about possibilities by the time they are 2 or 3 years old, but it is not clear what concepts they express with their modal vocabulary.

Learning to speak the language of possibilities may provide a workspace for developing modal concepts.

Why Study Modal Representations?

The origins of the human capacity for **logically complex representations** (see Glossary) have been the subject of *a priori* philosophical arguments at least since Descartes [1], who argued that only linguistically competent humans could entertain such thoughts (see also [2]). However, this question is not to be settled from the armchair. We need empirical studies that show which, if any, **logical concepts** nonlinguistic individuals deploy, and we must describe how these capacities develop, both over phylogenesis and ontogenesis. Such empirical work must proceed through case studies. Here, we consider the case of **modal concepts:** POSSIBLE, IMPOSSIBLE, and NECESSARY. Reviewing what is known about the acquisition of modal concepts is timely for at least two reasons. First, these concepts indubitably enter into logically complex thoughts. Not only are modal concepts logical concepts, but there are also clear relationships between modal concepts and other logical concepts, such as OR, between modal concepts and abstract concepts, such as PROBABILITY, and between modal concepts and metarepresentational concepts, such as UNCERTAINTY. Second, there has been an explosion of recent theoretical work on modality across the cognitive sciences (for philosophical overviews, see [3–5], for linguistics overviews, see ([6–9], A. Cournane, unpublished), and for empirical work, see [10–21]).

We begin by reviewing work that has been interpreted to show that prelinguistic infants and non-human animals create modal representations of possibility. These results fly in the face of evidence, which we review next, that even preschoolers fail to use concepts such as NECESSITY, POSSIBILITY, PROBABILITY, CERTAINTY, and UNCERTAINTY on related tasks. One tool for addressing this apparent conflict is a distinction between minimal representations of possibility and modal representations of possibility (Figure 1, Key Figure). The essential feature of the modal concept of possibility under investigation here is a symbolic marker that combines with a representation of a state of affairs (whether in the form of an iconic model or a sentence-like representation of a proposition) to express that that state of affairs is merely possible, as opposed to impossible or necessary or simply true in the actual world. Such a representation underlies the capacity to entertain multiple incompatible possibilities, each of which might be true or false of the actual world. The modal concepts of interest here underwrite the ability to tell the difference between what must be the case, what might be the case, and what cannot be the case.

Minimal representations of possibility contain no symbolic means for representing a possibility as such. All complex animals make predictions; predictions and **simulations** have crucial roles in perception and in action planning. Looking from the outside, the representations that are the output of prediction and simulation are merely possible; predictions can be and often are wrong. However, the predictor and/or simulator need not mark them as such and need not endow them with a **logical structure**. Box 1 discusses what might be achieved on the basis of minimal representations of possibility alone.

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Key Figure

Two Ways to Represent Things that Are Possible

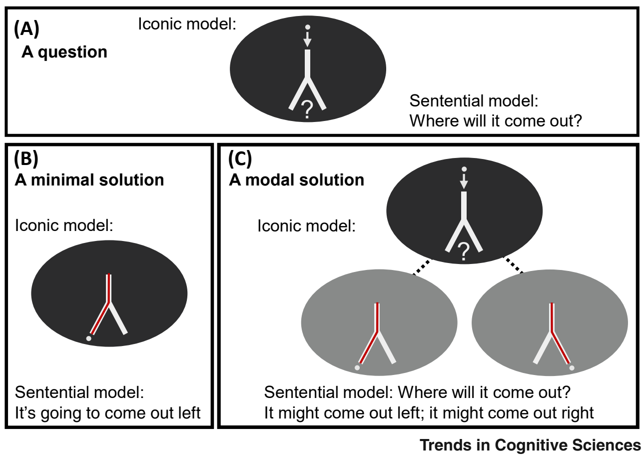

Figure 1. Black circles represent models of the actual world. Question marks represent open questions. Each panel contains an example of an iconic model and a parallel example of a sentential model. (A) A ball will soon be dropped into a Y-shaped tube [12], engendering the question, 'Where will the ball emerge?'. Two incompatible outcomes can be simulated: left and right. These cannot be part of the same model on pain of inconsistency. We describe two strategies for answering the question while preserving consistency in the model. (B) One might address the question by picking one simulation and assuming that it is actual. The model of the actual world, be it iconic or sentential, is updated with this information, and the question is taken to be answered. This is a minimal representation of possibility. (C) Participants who use modal representations of possibility mark representations with a symbolic operator. A function of this operator is to keep track of representations that cannot be ruled out but that also cannot be ruled in and added to the actual model. This can be accomplished in an iconic model by building a subsidiary model for each incompatible outcome, and marking it with a symbol (here, by coloring it gray) to indicate that it captures one answer to the question that cannot yet be ruled out but cannot yet be ruled in either. In a sentential model, the same function can be accomplished by adding a symbol such as the word 'might' to each of the subsidiary sentential models of the situation. Either way, the question remains unanswered in the model of the actual world. However, the subsidiary models limit the range of answers and the question is only partially answered.

Prelinguistic Humans and Nonlinguistic Animals Represent Possibilities

At least four sources of data have been taken to show that modal representations of possibility arise in nonlinguistic thought, in animals, and in infants before they master the language of modality.

Deductive Inference Involving Disjunction: Call's Cups Task

There is an intimate relationship between considering multiple possibilities and forming or holding disjunctive beliefs. Representing a disjunction is usually wasteful unless both disjuncts are possibly true and possibly false; indeed, some analyses of disjunction posit that the meaning of 'A or B' is the same as 'possibly A and possibly B' [22]. Thus, evidence that animals or infants reason through the disjunctive syllogism (A or B, not A, therefore B) is evidence for modal representations of possibility. Data from Call's cups task have been taken as such evidence. In this task, a desirable target is hidden in one of two occluded locations (A or B). Both A and B are possible locations for the target. If participants are shown that A is empty (not A), great apes, monkeys, birds, elephants, and many other nonlinguistic species immediately search in location B (therefore B; e.g., [23–31]). Twenty- and 23-month-old humans, who do not yet grasp the meanings of 'not' [32–35], 'or' [36,37], 'possible', or **modal operators**, such as 'could', 'might,' 'must', or 'have to' [38,39] also pass this task [34,40].

Glossary

Epistemic modals: can be characterized semantically or syntactically. Semantically, their meaning is sensitive to an epistemic state: 'Given what we have learned, the culprit must be Jones' rather than, for example, a set of rules: 'Given what the rules are, Jones must not park there'. Syntactically, epistemics scope above tense and aspect, unlike root modals.

Logically complex representation: structured representation including at least one logical concept.

Logical concepts: concepts that operate on representations of states of affairs with propositional content, thereby increasing the complexity of their logical structure. Examples of such concepts include those expressed with English words such as 'and', 'or', 'not', 'every', 'some', 'possibly', and 'necessarily', which attach to sentences. Operators with the same content may modify iconic models (Figure 1).

Logical structure: structure that underwrites deductive inference.

Modal concepts: symbolic, logical concepts that mark representations as necessary, merely possible, or impossible. For example, a representational system might have symbol that attaches to sentences and marks them as merely possible. Alternatively, a representational system might have some means of marking an entire iconic model as merely possible, as a candidate model of the actual that is not yet accepted as the correct model.

Modal operators: objects in formal systems that have been constructed for, among other things, modelling the behavior of modal concepts such as POSSIBLY and NECESSARILY. Modal logics model the inferential behavior of modal concepts that give representations logical structure. Nonetheless, the objects of study here are not the operators of modal logic, not least because there are many modal logics. Formal logic has additional goals, such as studying the properties of formal systems, that make it properly a branch of mathematics and, in many cases, those goals are orthogonal to those of

Box 1. Getting by with Minimal Representations of Possibility Alone

How would a creature who lacked modal representations of possibility but had minimal representations be impaired? Where would their performance be effective?

In contexts where possibilities can be considered and rejected in sequence, minimal representations of possibility will often work fine. For many goal-directed actions, such as a young chimpanzee searching for its mother, it will suffice to generate a guess and act on it. If the guess proves false, the chimpanzee can generate a new guess and repeat the process until the goal is met. Its guesses can be guided by the facts it knows about the actual world (e.g., how often its mother has been in various locations). This requires that the chimpanzee represents the spatial layout of the forest it lives in; that is, knowledge it needs for navigation, foraging, and many other purposes. It need not mark every location in the forest as a potential place where its mother might be found; that is, it need not build multiple, incompatible models of a single present reality. It can draw on a single model of reality (of the forest, and frequencies of past encounters with its mother) to generate a prediction of where its mother is. If it goes to that location, and finds it empty of its mother, it simply generates a new prediction and searches there.

Sequential guessing can effectively guide searches in a more abstract sense, such as searching to find the answer to a question (Where did you get that object?). The questioner presupposes you know the answer, and you can search your episodic memory for the context in which that object came into your possession. You may be satisfied with the first answer you generate, but if your interlocutor knows a reason why that answer cannot be right answer, and tells it to you, you can search again.

A creature with only minimal representations of possibility will struggle when multiple possibilities must be considered in parallel. For example, they will not be able to generate contingency plans that take multiple incompatible possibilities into account at the same time. If I take your order for coffee, but forget whether you asked for milk or cream, then I can cover my bases by bringing both milk and cream to the table. Repeatedly guessing will not generate this solution.

Should a creature who makes guesses and adds them to their model be surprised when their guesses turn out to be false? We think not necessarily. Our commitment is that they will not be more surprised when a guess turns out false than they are when a well-justified belief turns out false, since they are not distinguishing their guesses from their well-justified beliefs.

Deductive Inference Involving Disjunction: Object Identity Disambiguation

Recent eye-tracking data apparently lower the age of human success at the disjunctive syllogism to 12 months of age [10], before even the age of mastery of subpropositional combinatorial structure [41]. Infants watch videos of two objects (e.g., a snake and a ball, Figure 2A). The objects end up hidden in two places, one behind a screen and one in a cup; participants cannot tell which object is where. Thus, the cup contains either the snake or the ball. They then learn that the snake is not in the cup, enabling the inference that the ball is in the cup. A variety of measures suggest that infants do indeed make this inference (Figure 2).

These data are consistent with the conclusion that 12-month-olds are able to establish representations of two incompatible possibilities as to a current state of affairs, eliminate one of them when given evidence to do so, and conclude that the other must be true.

Uncertainty Monitoring

Awareness of multiple possibilities is one potential basis of metaconceptual representations of uncertainty. Prelinguistic infants and many non-human species will seek further information before acting when their current knowledge state is inadequate to the problem at hand [42–47], or will opt out if they are allowed to when they are uncertain, moving on to the next trial [48–55]. For example, if responding incorrectly on a trial incurs a large cost (e.g., waiting a minute for the next trial), but trials can be skipped at a smaller cost (waiting 20 s for the next trial), individuals from many species will skip difficult trials but not easy trials. If 20-month-old infants see a toy being put in one of two boxes, they will reach for it; but if the boxes are occluded so that they cannot tell which

cognitive science. Some research in cognitive science concerns which modal logic, if any, best captures the modal concepts that articulate ordinary language and thought; that is not our project here.

Root modality: modal verbs in a syntactic position between the verb phrase and tense and aspect phrases. An open question is how syntactic position is related to variance in the observed meanings of modal verbs.

Simulation: a process of generating new representations of outcomes from a model that idealizes, simplifies, or is less expensive than actual manipulation of the world. Simulations can be used to generate predictions, to calculate frequencies in possible outcomes, and so on. Simulations can be mental, as when we project ourselves into the future from a given point in time and space, or as when we reason counterfactually (e.g., about what might have happened if hurricane Dorian had hammered Boston, or about how things would be different if some entities we thought were people were actually robots or aliens). Simulations can also involve physical models, as when an engineer builds a model bridge to explore how it might break down under different distributions of the traffic crossing it, or different weather conditions.

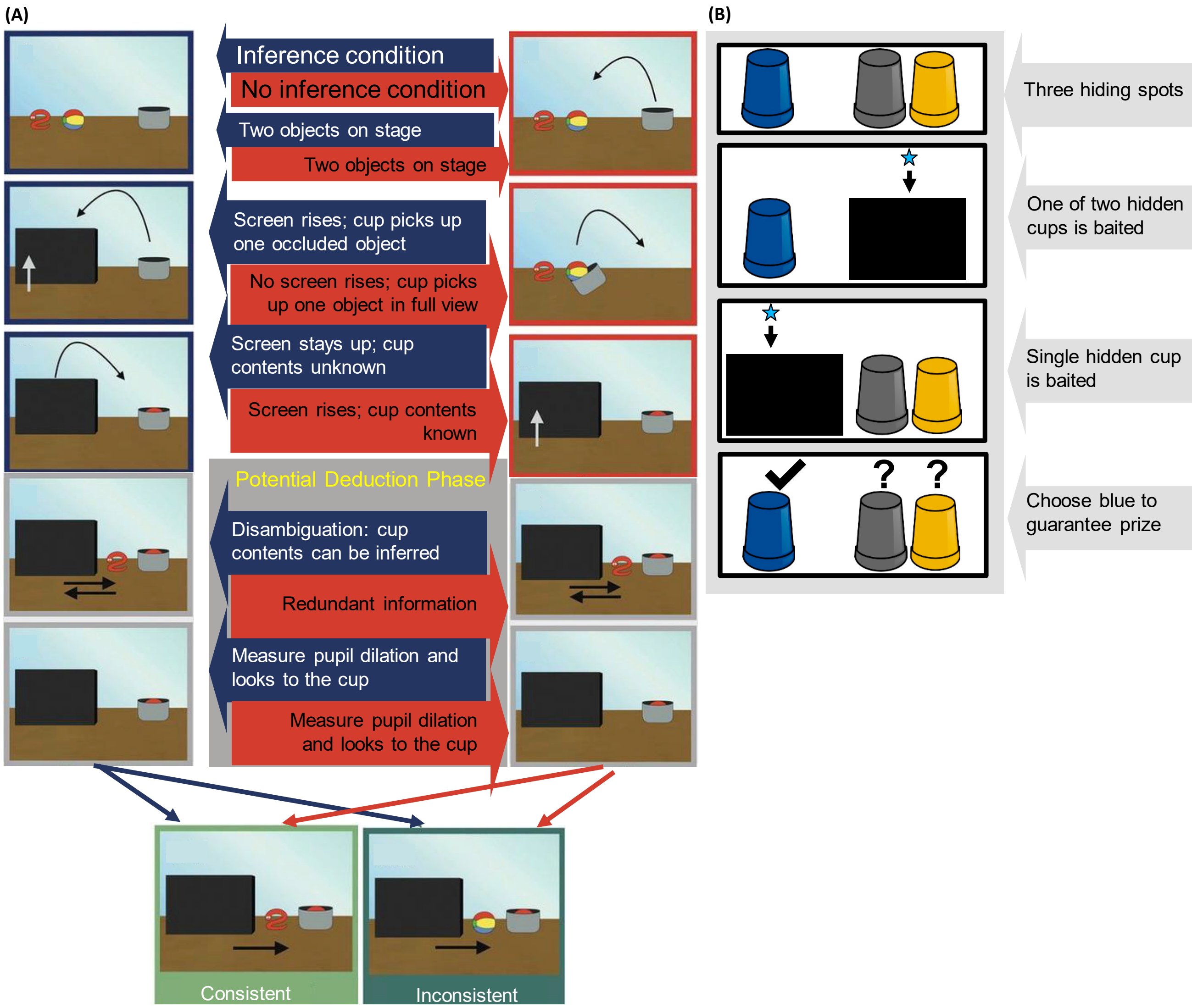

Figure 2. Two Conflicting Findings Regarding Children's Sensitivity to Possibilities.
(A) Object identity disambiguation. During the potential deduction phase, even 12-month-olds show increased pupil dilation and more looks to the cup when they receive disambiguating information, compared with when they already know what's in the cup. Are they inferring, from the fact that the snake is behind the occluder, that the ball is in the cup? (B) Distinguishing certainty from uncertainty. Three-year-olds and great apes choose the target cup ~50% of the time [40,58,59]. Why are participants better than chance (33%), but still so far from ceiling?

box the toy is hidden in, they will ask for help [56]. Even among infants who see where the toy is hidden, the proportion who ask for help increases as the time since the toy was hidden increases. This suggests that increasing uncertainty is modulating infants' decision to ask for help, consistent with modal representations of multiple possibilities.

Sensitivity to Probability Without Relevant Frequency Information

Human adults generate predictions or expectations for one-shot events in novel situations where frequency information is unavailable. So do 12-month-olds [17–20,57]. For example, if 12-month-olds see a ball bouncing inside a box that has three openings on one side and one opening on the other, their anticipatory looks show that they expect the ball to emerge from the side with more openings. Moreover, their looking times reflect surprise when the ball emerges from the side with one opening [17]. Both of these results suggest that infants expected the ball to emerge from the side with more openings. These expectations could be generated by calculating the proportion of possibilities in which the ball comes out each side. If this explanation is correct, the infant is performing a computation over representations of multiple incompatible open possibilities.

These four lines of research provide data from nonlinguistic tasks that are consistent with prelinguistic infants' considering multiple incompatible possibilities when making inferences and choosing actions. Additionally, in the case of uncertainty monitoring and Call's cups task, non-human animals provide evidence for representations with the same content. However, we now describe several abject failures on the part of young preschool children to display those same capacities, raising doubts that rich interpretations of the infant data are warranted.

Preschoolers Fail to Condition Responses on Representations of Incompatible Possibilities

Failures to Distinguish Certainty from Uncertainty

While 20-month-olds opt out (ask for help) in ways that take their uncertainty into account, toddlers, non-human animals, and preschoolers younger than age 4 often fail to reliably distinguish certainty and uncertainty when planning actions. For example, when a sticker is hidden in one of two occluded cups, and then a second sticker is hidden in a third occluded cup (Figure 2B), then the first sticker could be in either of the first two cups, while the second sticker is certainly in the third cup. When given just one chance to obtain a reward, chimpanzees (where the reward is desirable food) and children younger than 4 pick the certain cup only about half the time [40,58,59]. This performance is not completely random; since there are three locations, chance is 33%. However, performance is still poor. Four- and 5-year-olds choose the target >50% of the time, but performance is still far from adult levels of 100% choice of the certain cup.

Modal representations of two open possibilities on the two-cup side would support distinguishing certainty from uncertainty, probabilities of 1 from probabilities of $^1/_2$, both abilities claimed for infants on the basis of the experiments discussed above. Thus, failure in this simple task should raise doubt about the interpretation of the infant studies as reflecting representations of incompatible possibilities.

Failure to Distinguish Partial from Total Ignorance

In many conditions of uncertainty preschoolers appear to commit to a guess, and they appear to confuse their guesses with knowledge until after age 4. When simply shown a small box and asked, 'Do you know what's in here, or do you not know?', even 3-year-olds say they do not know (total ignorance trials). However, on partial ignorance trials, the experimenter first showed the participant two toys (e.g., a lion and a tiger), and explained that he was hiding one of them in the box. Partial ignorance trials were harder, with 3- to 5-year-olds usually saying they knew, and telling the experimenter which item was in the box (e.g., 'a lion') [46,60,61]. They guessed about the contents of the box, and treated that guess as knowledge. They failed to condition their response on having represented the lion and the tiger each as alternative possible candidates for the content of the box. Marking each option as merely possible should entail uncertainty.

This task is pragmatically odd in two ways. First, when we ask somebody if they know something, we usually are asking them to tell us, not to reflect on their state of knowledge (consider, 'do you know what time it is?'). That is, the intended interpretation of the question is metaconceptual: children are being asked to reflect on their state of knowledge, which may be difficult for preschoolers. Second, the experimenter knows what is in the box; the child might think he or she is being asked to guess. All of these problems with the task are resolved by changing the procedure: a new experimenter enters the scene and wants to know the contents of the box. Participants may either inform them about the contents of the box or opt out, allowing the first experimenter to inform them. This enabled more 4-year-olds to acknowledge their ignorance on partial ignorance trials, but not 3-year-olds [61], and even 4-year-olds' performance was not yet adult-like. This converges with the results on the 3-cup task (Figure 2) that 4-year-olds are beginning to show signs of modal representations of possibility.

Failure to Prepare for Multiple Alternative Possibilities

If a desirable target is dropped into a tube made of PVC pipe shaped like an upside-down Y (see Figure 1 and Figure I in Box 1 in [62]), participants can easily catch it if they cover both arms of the Y. Two-and a half-year-olds and great apes almost universally only cover one arm, even after many trials (up to 96 trials for great apes), thus missing the target half of the time [12,62]. Children start using two hands around their third birthdays, but without evidence of insight, for success is not spontaneous, and 3-year-olds (and apes) often regress to using just one hand even after trials in which they did cover both arms. By their fourth birthdays, most children used both hands from the first trial.

This task reveals a striking failure by apes and by children under 4 to condition action on two incompatible possibilities.

Summary

Adult performance on the Y-shaped tube, partial ignorance, and 3-cups tasks requires more than generating a prediction, acting on it, and revising it upon gaining inconsistent information; adult performance requires representations that go beyond minimal representations of possibility. One must realize that there are two incompatible possibilities that are open about a single future (Y-shaped tube) or current, but unknown (partial ignorance, cups task), state of the world. That is, success on these tasks requires modal representations of possibility, representing possibilities as such: that is, possibly A and possibly B.

Resolving the Infant's Successes and the Preschooler's Failures

The previous two sections showed conflicting results: on the one hand, there are several tasks that infants and nonlinguistic animals pass that appear to demand taking possibilities into account. On the other hand, several apparently simple tasks requiring the same abilities are difficult for children until around age 4. We see two strategies for reconciling this conflict. First, the tasks posed to preschoolers may underestimate their capacity for modal representations. For example, preschoolers might fail on the described tasks because of performance demands that are independent of a capacity for modal representations of possibility that they in fact have. Second, animal and infant successes could be being overinterpreted; perhaps simpler strategies can account for their performance. We believe the second resolution is most likely; that is, we suggest that non-human animals and young preschoolers can form only minimal representations of possibility. We offer a parsimonious account of the infant successes and the details of the preschooler's failures. Such a parsimonious account is a warrant for accepting this hypothesis. We discuss these two routes to reconciliation in turn.

Task Demands Mask Preschool Competence

Maintaining explicit representations of several possible states of affairs, and explicitly evaluating the evidence for each, certainly makes executive function (EF) demands on working memory, on shifting among possibilities, and inhibition of the possibility not currently under consideration. EFs develop markedly throughout all of childhood, with rapid and dramatic change in 3- to 5-year-olds [63,64]. Perhaps limitations in EFs account for preschoolers' shortcomings. We have no doubt that this is part of the problem, but doubt that it is the whole story.

First, the rich interpretation of the infant data as revealing access to logical concepts such as OR or POSSIBILITY requires that infants can maintain working memory models of multiple possibilities, marked as such, and have the capacity to selectively consider each in inference and update their representations in working memory. Thus, on this rich interpretation, those tasks also make high working memory, set-shifting, and inhibition demands, and infants are even more deficient in EFs than preschoolers. An appeal to performance limitations as the source of failures in the preschool years is empty in the absence of testable hypotheses about the performance demands of the preschool task, which are absent in the infant task.

Second, it is not obvious that the 3-cups task exceeds the executive function capacity of 3-year-olds. Even 10-month-olds can set up two working memory models, provided they are models of different situations. They can track up to about three items in a single set, and they can track multiple sets, shifting attention between them to decide which set is larger. If an experimenter puts three crackers in one bucket and two crackers in another, they will reliably crawl toward the bucket with more crackers, even though they cannot track five items with a single model [65,66]. That is, infants and toddlers have sufficient executive functions to maintain two models at once, each of which models for different aspects of a structured representation of the actual world (what is in A and what is in B), and selectively attend to one of them in the service of inference or action planning. However, these models are not alternative competitors for a single reality that the child cannot yet distinguish between. They are models of two distinct situations, not alternative models of a single situation. Although they are not logically complex representations of two possibilities, these abilities require two models to be held in working memory, distinguished from each other. Relatedly, there is also evidence that even infants can maintain a model of reality and a model of what another person thinks is reality [67–70], and toddlers can maintain a model of reality and a model of pretend reality [71,72]. These abilities also require two models to be held in working memory: reality versus what that agent represents as reality, or real world versus pretend world, respectively, and, thus, would appear to tax executive functions as much as the tasks that preschoolers struggle with. The problem for contingency planning on the Y-shaped tube task, for knowing that one does not know that it is the lion on the partial ignorance task, and for picking the certain cup on the 3-cups task, lies with maintaining incompatible models of a single situation at the same time, a feat that requires a logical concept that marks representations as merely possible.

We cannot fully rule out the hypothesis that the Y-shaped tube, 3-cups, and partial ignorance tasks make excessive domain general processing demands. Testing this hypothesis would require specifying what domain general processing capacities are deficient and seeking evidence that differences in those capacities predict variance on the tasks. Such detailed hypotheses are currently lacking. Training studies in which those capacities are strengthened could be mounted, and if merely lifting the domain general limitation is driving the developmental change, successful training of that capacity should yield better performance on the tasks.

Consequences of Working with Minimal Representations of Possibility

We propose that nonlinguistic animals and humans through the early preschool years have the capacity for minimal representations of possibility alone. They cannot compare the results of incompatible simulations but can use simulation to generate a single result and treat that result as reality (Figure 1). Moreover, they can revise their model in the light of evidence that it is inadequate (Box 1). This proposal explains the pattern of successes and failures we reviewed earlier.

Call's cup task is a canonical search task explainable by minimal representations of possibility. Upon seeing the two cups revealed after the hiding event, infants may simulate the prize in one of the cups. If the experimenter then shows that cup to be empty, they simply revise this guess and generate a new one, namely the other cup, which is where they search.

The object disambiguation data can also be explained by single simulations, plus revision. If participants in the inference condition simply guess which object is in the cup, then, when disambiguating information arises in the potential deduction phase as one object emerges from behind the screen (Figure 2A), about half of the participants will need to revise their guess, reflected in looks to the cup, and requiring cognitive effort, hence pupil dilation. Neither of these is required in the no-inference condition, because the child has already seen which object is in the cup.

In the one-shot probability experiments, the group-level results can be explained by each infant on each trial simply running a single simulation, guessing which exit the ball will come out of at random. If so, on three-quarters of the trials, infants will anticipate that the ball will emerge from the more likely side, and will be surprised that the ball exited the less likely side. Conversely, on a quarter of the trials, infants will anticipate that the ball will exit the less likely side, and show surprise when the ball emerges from the likely side. This is the observed pattern of results.

Next consider uncertainty monitoring. Why do 20-month-olds ask for help when they are uncertain but not when their knowledge is adequate to the task? Why do animals opt out more and gamble less on their accuracy when their certainty should be lower [48–55], and seek more information when uncertain [42–47]? One possible answer to this question is that running a single simulation and adding the results to the current model of reality is a last resort. Animals and children may prefer first to look for more information, or prefer not to act, if these are options. Perhaps it is only when circumstances require action that children under age 4 generate a prediction and treat that representation as reality (see Box 2 for further discussion of this issue).

The performance of 2.5-year-olds and apes on the Y-shaped tube task transparently reflects guessing where the ball will exit and acting on that guess. Similarly, the failures on the partial ignorance task is transparently consistent with the children simulating one of the two objects being hidden in the box and treating the output of that simulation as knowledge.

Finally, this proposal predicts not only that participants will struggle with the 3-cups task (Figure 2B); but also the details of the observed distribution of responses. Chimpanzees and 2- and 3-year-olds choose the certain cup 50% of the time, better than the 33% that would be expected if they were choosing entirely at random. A creature with minimal representations of possibility alone knows that there is a reward in the single cup. When the reward is in one of two cups, they can simulate to

Box 2. Possibility and Uncertainty

Uncertainty and Modal Representations of Possibility

The existence of multiple open possibilities, represented as such, should engender explicit beliefs concerning whether any one is true. When I recognize that there are multiple open possibilities, I am unwarranted in being certain that any one of them is true, and may explicitly represent my uncertainty as a result.

Uncertainty in the Absence of Modal Representations of Possibility

Uncertainty can guide action among those who cannot mark representations as merely possible. Animals and children under four decide to seek more information [42–47] and to opt out of tasks when errors are costly [48–55] in ways that covary with their uncertainty. Nevertheless, this does not guarantee that animals and young children are aware of their uncertainty (see Box 3 in [62]) or that they have modal representations of possibility. Properties of representations such as contrast and size engender illusions of certainty: animals and humans are willing to take more chances and gamble more in match-to-sample games when the samples are larger or drawn in higher contrast, even when these factors do not impact accuracy [94–100]. Young children can be trained to identify behaviors that covary with uncertainty and use them as clues when answering questions about their own uncertainty [101–103]. How are these facts to be explained?

Representations are physical states of the nervous system. Mental processes might respond to various measures of the quality of representations to modulate behavior, reducing confidence when faced with features of representations that are correlated with uncertainty. Identifying behaviors and perceptual features of represented states of affairs that covary with uncertainty does not require an explicit concept of uncertainty. Creatures with minimal representations of possibility alone may identify when to seek more information and/or when to opt out, by learning correlations between those behaviors and perceptual features and the probability of success on goal-directed tasks. The evidence that animals and children (and adults) monitor certainty shows that they are sensitive to cues to uncertainty (hesitation in making a choice, degrees in vividness of the representation, etc.), and rely on them sometimes when deciding to seek more information or opt out. Such procedures do not require an explicit concept of uncertainty that is related, as it is sometimes in adults, to modal representations of possibility.

establish a guess about which cup it is in and treat that guess as fact. If this creature does not mark the latter simulation as a mere possibility, then, from its point of view, it knows where both rewards are hidden. Thus, it chooses randomly between them, generating the observed 50% choice of the certain cup.

Minimal Representations of Possibility Alone under Age 4?

In sum, there are cohesive, uncomplicated explanations that account for the successes in the infant studies and the failures in the early preschool years that presuppose minimal representations of possibility and only minimal representations of possibility, before about age 4. However, even for this hypothesis, we must account for what representations underlie single simulations. The generation of even a single guess can be thought of as a Bayesian choice over multiple hypotheses (possibilities) that are weighted by prior probabilities. Revision in this context can be modeled as adjusting priors in the light of new evidence (posteriors). We endorse this view. Box 3 sketches ways of thinking about probability distributions that do not require modal representations of possibility.

In sum, we argue that the hypothesis that children have access to minimal representations of possibility alone is consistent with the data we have reviewed here: both the successes of children under four, including infants, and the failures in the early preschool years.

Acquiring the Language of Modal Concepts

Language expresses modal concepts in many ways, in **modal auxiliaries** (e.g., 'might', 'must', 'can', or 'can't') and other words ('maybe', 'possibly', 'necessarily', or 'impossible') [6]. If the modal concepts

Box 3. Possibility, Probability, and Decision Making

Representations of possibility are closely connected to representations of probability [73–78]. Does the success of Bayesian models of learning, perception, and decision making [20,79–81] in all creatures, including animals and infants, commit cognitive science to the existence of modal representations of possibility in nonlinguistic thought? After all, Bayesian models require a space of hypotheses (possibilities) with prior probabilities and procedures for updating priors upon encountering new evidence.

Probability and Modal Representations of Possibility

When probability distributions are generated, at least in part, by counting up frequencies among simulated possibilities [17,18,57], then probability distributions require modal representations of possibility.

Probability and Minimal Representations of Possibility

However, probability distributions can also be generated by counting observed frequencies and proportions [82–87]. Representations of frequencies are not representations of possibilities as such, but rather summaries of multiple actual observations or proportions in a population or sample. Summary representations of past frequencies can guide behavior in a manner consistent with Bayesian principles. For example, suppose I need to find my keys before I leave the house. I have seen my keys on the hall table more often than I have seen them in the sugar bowl; this information can guide my predictions about where I will find my keys. A single prediction can be drawn from the range of places where I have seen my keys in the past, weighted by relative frequency (*cf.* [88]); this will more likely cause me to predict that my keys are on the table than that they are in the sugar bowl.

When an action based on that simulation does not work out, a new simulation can be generated (see Box 1). Iterating this process will reflect the probability distribution [89–91]. This process does not require the actor to manage incompatible possibilities, each marked as merely possible, although the behavior will be guided by probabilities.

Thus, hypothesis spaces can exist without modal representations of possibility, even though a theorist might naturally think of each competing hypothesis as a possibility. Importantly, probability theory may not provide a useful way to mark a representation as merely possible in contrast to fully accepted. If we say that a hypothesis is accepted if it has probability 1, then all alternatives to an accepted hypothesis must have probability 0, and so none are merely possible. However, if we set the bar lower than one, we encounter paradoxes: I might accept that my keys are somewhere in my house while denying, for every location in my house, that the keys are there (the lottery paradox [92,93]).

Box 4. Acquisition of Modal Language

Natural languages mark modality in many different ways. Much literature in the acquisition of the linguistic expression of modal concepts concerns the acquisition of different flavors or classes of modals: that is, **root modals** and **epistemic modals**. However, the concepts of concern here cut across that distinction. That is, both root modals and epistemic modals express modal concepts of possibility, necessity, and impossibility.

Corpus studies show that even 2-year-olds use modal language [104,105] and that many linguistic devices that express modal concepts in mature language are attested in child speech by age 3 [38,106–110]. Moreover, it is hard to tell from production studies what concepts a child expresses with their utterances. For example, the 'maybe' in 'Maybe my daddy give me a big piano' might simply mark that the embedded proposition is part of a pretend world, one that is not a candidate for actuality. It might mark the child's expectations for the future, without any awareness that her daddy might not give her a piano. It might be a request or an expression of the desire for a piano. All of these alternatives are consistent with the young speaker having only minimal representations of possibility.

Specifying what the child takes modal language to mean requires comprehension studies. Existing comprehension studies have found mixed results in young children. Suppose a marble is drawn at random from a container containing one red and ten blue marbles. Four-year-olds correctly say that the marble could be red, could be blue, but cannot be yellow [15]. However, 4- and 5-year-olds also respond at chance to simple modal questions such as 'Can the toy be in the yellow box?' when a toy had been hidden, under occlusion, in one of two boxes [21]. There are as yet no comprehension studies of this sort that reveal success among children age 3 and under.

Four- to 6-year-olds tend to say that immoral events (including events that should be familiar, such as lying and fighting) are impossible [16]. Four-year-olds fail to distinguish the improbable from the impossible: they are as likely to say that unlikely events, such as encountering a polka-dotted airplane, could not happen in real life as they are to say that impossible events, such as eating lightning for dinner or walking through a wall, could not happen in real life [15]. Their struggles persist if the question is asked without overt modals: 'Would it take magic for there to be pickle-flavored ice cream?' They answer yes, and to the same extent as if asked whether it would take magic to walk through a wall. Learning modal language is difficult and protracted, and current evidence reveals only the beginnings of adult-like representations underlying the language of modality by age 4.

that support meanings of these linguistic expressions are not available to children below age 4 or so, 2- and 3-year-olds should not be able to comprehend the natural language expressions that encode the language of modality. Box 4 points to the current evidence that children do continue to struggle with the language of modality at least up until age 6. Thus, nonlinguistic tasks suggest that modal concepts begin to condition behavior around age 4, and studies of understanding modal language suggest that children also begin to grasp the adult meanings around age 4. However, what is the relationship between the acquisition of modal concepts and learning how modal concepts are expressed in language?

It still is an open question whether modal concepts must be acquired at all. Perhaps they are innate, and the findings we have outlined in this article arise because, during development, children develop different attitudes about how and when to use modal concepts and how and when to talk about them. In terms of this proposal, learning modal language is just a mapping problem of figuring out which words in the language go with which concepts. As both this piece and the companion piece insist, a high priority for future research is to seek positive evidence for modal concepts in nonlinguistic thought; without such evidence, we do not consider this hypothesis further.

Alternatively, as we have argued here, modal concepts might arise through some process of conceptual construction. We see two different broad distinctions between the ways in which this construction might proceed that could account for the relations between age transitions in success on the nonlinguistic tasks and in the modal language studies.

First, the construction process might be independent of language, taking place before assigning adult meanings to modal language. In this issue of *Trends in Cognitive Sciences*, Redshaw and

Outstanding Questions

What are the mechanisms of change from minimal representations of possibility to modal representations of possibility? Clues are already available from the relationships between linguistic and nonlinguistic tasks, but concrete proposals are yet to be formulated.

Failing the 3-cups task, partial ignorance task, and Y-shaped tube suggests an absence of modal concepts, but what do successes show us? Griffin, an African gray parrot, demonstrated that mastery of the language of modality is not necessary for success on the 3-cups task [30]. Does this show us that modal concepts are possible in the absence of language?

What sources of evidence would establish that modal concepts articulate nonlinguistic thought? Can nonlinguistic probes for the concepts NECESSITY and IMPOSSIBILITY be designed? Evidence that animals or infants distinguish necessity or impossibility from mere possibility would provide convincing evidence for nonlinguistic concepts of possibility.

Although corpus work shows that 2 year-olds use words such as 'maybe', what do comprehension studies or elicited production studies tell us about children's understanding of 'necessary', 'merely possible', and 'impossible'? More work on the meanings young children from ages 2 through 6 actually assign to modal auxiliaries and other modal vocabulary is needed.

Is there within-subject and between-task consistency across the nonlinguistic tasks taken to diagnose modal concepts and understanding modal language? Can training and/or priming studies eliciting modal language improve performance on the nonlinguistic tasks? Answers to these questions would bear on the mechanisms through which both modal concepts and modal language are acquired, as well as the relations between these two processes.

Suddendorf [62] suggest that modal concepts require metaconceptual developments that take place late in the preschool years, and metaconceptual reasoning may have an important role in the construction process. In terms of this view, acquiring modal language would largely be a difficult mapping problem, and this cannot begin until there are nonlinguistic modal concepts for language to map onto.

What computations go into evaluating modal propositions, and how might these change with age, independently of the capacity to represent possibility as such?

Second, the construction of modal concepts might not be independent of language. The data sketched in Box 4 are important in this context. The fact that even 2-year-olds are using modal language in ways that reflect its frequency in the speech they hear shows that they can identify contexts in which it is appropriate (even if it is not yet encoding adult modal concepts). Perhaps learning modal vocabulary and the relationships between various modal words provides a workspace that helps children construct appropriate meanings for those words and, hence, the concepts themselves.

Concluding Remarks

The pattern of infant and animal successes on tasks related to modality contrasts strikingly with preschoolers' failures on other simple modality tasks. This conflict can be resolved by distinguishing modal representations of possibility from minimal representations of possibility. Modal representations of possibility are logically structured. They use a symbolic operator whose function is to mark representations as merely candidates for actuality, members of a set of alternatives that cannot yet be ruled out but cannot be ruled in yet either. Minimal representations of possibility are guesses about what the world is or will be like, guesses that are guided by representations of what the world is actually like, including frequency information about how the world has been in the past. Minimal representations of possibility need not have any logical structure. Individuals who do not have access to modal representations of possibility can still solve many problems by using minimal representations of possibility.

Both the present review and its companion piece [62] argue for a transition from an absence of the capacity to represent multiple incompatible possibilities in non-human animals and children under age 4 to the presence of such a capacity. Neither piece explains the mechanisms of this transition. Studies that address such questions (see Outstanding Questions) will advance our understanding of the acquisition of the capacity for thoughts that are logically structured by modal concepts, and will bear more generally on Descartes' question of the relations between language and abstract combinatorial thought.

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