Exercises on some game theoretical notions

1 Normal forms of games

1.1

V and A are two brothers: V wants to play either with the train or with the Legos, but he wants to play alone. A also wants to play with the train or with the Legos, but he wants to play with his brother V. Express this strategic situation formally in a normal form.
Hint: You can use arbitrary numbers for expressing utilities as long as their relations express the preferences specified in the text.

1.2

Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive.

1.3

In Charness and Rabin's 2002 paper (Understanding Social Preferences with Simple Tests), participants to the experiment are asked to make the following choice:

> Participant A chooses a monetary distribution of 750 Monetary Unit (MU) for himself and 0 to B, or lets B chooses between two distributions: either 400 MU for each, or 750MU for A and 400MU for himself.

Write the extended form and the normal form of this game (assuming that the monetary payoffs are the only relevant incentives).

2 Finding Pure Strategy Nash Equilibria

Please circle the pure strategy Nash equilibria in the games below. Remember that a game can have no pure strategy Nash equilibrium or more than one.[1] If a game is familiar to you (e.g. Prisoner's Dilemma or Stag Hunt), please write down his standard name.

2.1

2, 2	0, 0
0, 0	1, 1

2.2

1, 1	1, 0	0, 1
1, 0	0, 1	1, 0

2.3

2, 2	1, 3	0, 1
3, 1	0, 0	0, 0
1, 0	0, 0	0, 0

[1] For your convenience, here is a definition of a Nash equilibrium:
Let $\{a_i\}_{i\in I}$ be a set of strategy profiles for each agent i belonging to I for a given game.
Let $U_x(\{a_i\})$ be the utility of x when each players of I play the strategy a_i.
$\{a_i\}_{i\in I}$ is a Nash equilibrium for this game iff

$$\forall x \in I, \quad \forall a'_x, \qquad U_x(\{a_i\}_{i\in I-x} \bigcup \{a'_x\}) \leqslant U_x(\{a_i\}_{i\in I})$$

2.4

1, 1	0, 1	0, 0
1, 0	2, 1	1, 2
0, 0	1, 1	2, 0

2.5

Circle the Nash equilibria, if any, for the game played in 1.1

2.6

Circle the Nash equilibria, if any, for the game played in 1.2

2.7

Selfish and altruistic social behavior Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.

1. Suppose that each person cares only about her own comfort. Model the situation as a strategic game. Is this game the Prisoner 's Dilemma? Find its Nash equilibrium or equilibria.

2. Suppose that each person is altruistic, ranking the outcomes according to the other person's comfort, but, out of politeness, prefers to stand than to sit if the other person stands. Model the situation as a strategic game. Is this game the Prisoner 's Dilemma? Find its Nash equilibrium or equilibria.

3 Risk and expected utility

3.1

Gyuri is going to play an Ultimatum game with an anonymous player. He does not know who the anonymous player is, but he knows that the player is coming from his

own village. As a consequence, he can estimate the social preferences of the player. They are 1000 people in his village (without counting Gyuri), and he estimates that among those, 400 are inequity averse and 600 have no other-regarding preferences (i.e. they do not care about what others' material payoffs are, they just want to maximise their own material payoff).

The inequity aversion of the 400 people can be expressed as $\rho = 0.2$ and $\sigma = -0.35$ in Rabin and Charness' expression of inequity aversion:

If $\pi_B \geqslant \pi_A$ (I have more money or an equal amount) then

$$u_B(\pi_A, \pi_B) = (1 - \rho)\pi_B + \rho\pi_A$$

If $\pi_B < \pi_A$ (I have less money) then

$$u_B(\pi_A, \pi_B) = (1 - \sigma)\pi_B + \sigma\pi_A$$

Gyuri has no other-regarding preferences and just wants to maximise his expected material payoffs. The ultimatum game is framed such that Gyuri has to choose among the following possibilities:

1. Offer 1, keep 9
2. Offer 3, keep 7
3. Offer 7, keep 3

Which option will Gyuri choose? Explain.

4 Mixed Nash equilibria

Find all Nash equilibria in the following game:

	L	R
U	5, 6	0, 10
D	4, 4	2,2

Hint: you need to specify with what probability the column player will play L and with which probability the row player will play U, i.e. there are two positive numbers p and $q < 1$ to find out.