

## Solutions to the first homework exercise

**Tamas: the exercise on risk aversion**

In this exercise, you had to compare the expected utility of planting wheat with the expected utility of planting corn. The outcome of each action is monetary gain.

Utility for planting wheat:

$$\frac{2}{3}u(\text{€}2400) + \frac{1}{3}u(\text{€}1800)$$

Utility for planting corn:

$$\frac{2}{3}u(\text{€}1200) + \frac{1}{3}u(\text{€}4200)$$

You are told about the utility  $u(x)$  that Tamas derives from monetary gain  $x$ .

**If Tamas is risk neutral** The utility of monetary gain is a linear function of the monetary gain.<sup>1</sup> When comparing the expected utility of monetary gains, it is thus convenient to use the function  $u(x) = x$ .

The first part of the exercise therefore consists in comparing

$$\frac{2}{3}2400 + \frac{1}{3}1800 \quad \text{and} \quad \frac{2}{3}1200 + \frac{1}{3}4200$$

This turns out to be equal.

The expected utility of planting wheat is equal the expected utility of planting corn, so Tamas is indifferent between the two option.

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<sup>1</sup>A linear function is a function of the form  $f(x) = \alpha \cdot x + \beta$  with  $\alpha, \beta \in \mathbb{R}$

**If Tamas is risk averse** We cannot any more use this simplification  $u(x) = x$ . As said in the text of the exercise, saying that Tamas is risk averse is the same thing as saying that the marginal utility he derives from monetary gain is decreasing. In other words, he enjoys getting more, but for a similar increase of monetary gain, he derives more utility when he initially has little than when he initially has a lot. More generally, for all monetary gain  $x > 0$  and for all initial states  $y_1$  and  $y_2$  such that  $0 < y_1 < y_2$ , we have:

$$u(y_1 + x) - u(y_1) > u(y_2 + x) - u(y_2)$$

We can easily apply this equation to the expected monetary gains<sup>2</sup> of Tamas .

$$u(1200 + 1200) - u(1200) > u(2400 + 1200) - u(2400)$$

We then add  $u(1200 + 1200) - u(1200)$  to each side of this inequality.

$$\begin{aligned} 2[u(1200 + 1200) - u(1200)] &> u(2400 + 1200) - u(2400) + u(1200 + 1200) - u(1200) \\ &> u(3600) - u(1200) \\ &> u(1200 + 2400) - u(1200) \end{aligned} \quad (1)$$

Applying again the definition of risk aversion, we have:

$$\begin{aligned} u(1200 + 2400) - u(1200) &> u(1800 + 2400) - u(1800) \\ &> u(4200) - u(1800) \end{aligned} \quad (2)$$

The inequalities (1) and (2) give:

$$2[u(2400) - u(1200)] > u(4200) - u(1800)$$

which is equivalent to:

$$2u(2400) + u(1800) > u(4200) + 2u(1200)$$

Dividing each side of the inequality by 3, we obtain:

$$\frac{2}{3}u(2400) + \frac{1}{3}u(1800) > \frac{2}{3}u(1200) + \frac{1}{3}u(4200)$$

This is exactly the relation we were looking for. It says that:

expected utility for planting wheat > expected utility for planting corn

Therefore, if Tamas is risk averse, then he will decide to plant wheat. (By the same reasoning, we should be able to assert that if Tamas is risk seeking, then he will prefer to plant corn.)

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<sup>2</sup>for convenience, I did not write “€” in the arguments of the utility function.