

BUCKLE'S LAWS AND MAXWELL'S DEMON

Statistical law had been presented to the world by Quetelet and Buckle as proof that disorder and chance were epiphenomenal. The application by Maxwell of that most remarkable law of unreason, Quetelet's error curve, to the otherwise intractable problem of molecular velocities implied no vindication of what was still almost universally regarded as an unthinkable and self-contradictory illusion of untrained minds, objective chance, but an impressive extension of the domain of scientific order. The combinatorial operations it made possible were both rigorous and elegant, and it was no more to be anticipated that the reduction of thermodynamic propositions to mechanical ones would introduce a new element of uncertainty into the subject than that the application of number would have such an effect on social science. The idea that macroscopic regularities such as the second law of thermodynamics are only probable was manifestly a development of certain ideas associated with the statistical approach, but it involved at the same time a repudiation of the statistical viewpoint presented by Quetelet and Buckle.

That Maxwell should play a major role in the reinterpretation of statistical reasoning was fully consistent with his character and commitments. He was, of course, a physicist of exceptional creativity, revealed in work on electricity and magnetism that began in 1854 no less clearly than in his papers on gas physics. He was always eager to look at old truths from new perspectives, and in his inaugural lectures at Aberdeen in 1856 and King's College, London, in 1860 he warned his students against "assuming that the higher laws which we do not know are capable of being stated in the same forms as the lower ones which we do know."<sup>1</sup> A deeply religious man, Maxwell was sensitive to the limitations of natural science. His philosophy professor at Edinburgh, Sir William Hamilton, had observed that since the Deity could not be subject to necessity in the material universe, the study of nature could never attain more than "probable certainty,"<sup>2</sup> and although Maxwell's enthu-

<sup>1</sup> Maxwell, "Inaugural Lecture at Aberdeen, 2 Nov. 1856," *Notes and Records of the Royal Society of London*, 28 (1973), 69-81, p. 78; also "James Clerk Maxwell's Inaugural Lecture at King's College, London, 1860," *American Journal of Physics*, 47 (1979), 928-933, p. 930.

<sup>2</sup> Sir William Hamilton, *Discussions on Philosophy and Literature* (New York, 1856), p. 41; also pp. 275, 297. On Maxwell and Scottish Common Sense, see George Elder Davie, *The Democratic Intellect: Scotland and her Universities in the Nineteenth Century* (Edinburgh, 1961); Richard Olson, *Scottish Philosophy and British Physics, 1750-1880* (Princeton, 1975).

siasm for Hamilton waned after he left Edinburgh for Cambridge in 1850, he always took care to avoid implicating the scientific temperament as an obstacle to religious belief. "I have endeavoured to show that it is the peculiar function of physical science to lead us to the confines of the incomprehensible, and to bid us behold and receive it in faith, till such time as the mystery shall open."<sup>3</sup>

Maxwell was not persuaded that the bounds of the incomprehensible were being pushed back quite as rapidly as some of his contemporaries would have it, and he was troubled by the extravagant pronouncements sometimes made in the name of science. Prominent among the writers whose necessitarian claims bothered him was Henry Thomas Buckle. Maxwell had been highly impressed by certain aspects of Buckle's first volume and mentioned Buckle to his friend and subsequent biographer Lewis Campbell within a few months of its appearance: "One night I read 160 pages of Buckle's *History of Civilization*—a bumptious book, strong positivism, emancipation from exploded notions and that style of thing, but a great deal of actually original matter, the true result of fertile study, and not mere brainspinning."<sup>4</sup> Evidently Buckle set him, like so many of his fellows, to thinking about the relation between the remarkable regularities of statistics and free will, for three months later he wrote to his friend R. B. Litchfield:

Now, I am going to put down something on my own authority, which you must not take for more than it is worth. There are certain men who write books, who assume that whatever things are orderly, certain, and capable of being accurately predicted by men of experience, belong to one category; and whatever things are the result of conscious action, whatever are capricious, contingent, and cannot be foreseen, belong to another category.

All the time I have lived and thought, I have seen more and more reason to disagree with this opinion, and to hold that all want of order, caprice, and unaccountableness results from interference with liberty, which would, if unimpeded, result in order, certainty, and trustworthiness (certainty of success of predicting). Remember I do not say that caprice and disorder are not the result of free will

<sup>3</sup> Maxwell, "Aberdeen Lecture" (n. 1), p. 78. On Maxwell's relationship to Hamilton, and on his life generally, see C. W. F. Everitt, "Maxwell's Scientific Creativity," in Rutherford Aris et al., eds., *Springs of Scientific Creativity* (Minneapolis, 1983), pp. 71-141.

<sup>4</sup> Maxwell, pp. 294-295.



(so called), only I say that there is a liberty which is not disorder, and that this is by no means less free than the other, but more.<sup>5</sup>

No less objectionable to devout and conservative men like Maxwell were the public lectures and popular writings of the Victorian scientific naturalists. Maxwell seems to have gotten along well with T. H. Huxley, but there is ample evidence that he found John Tyndall's ideas wrongheaded and disagreeable. In a commentary inspired by some remarks of P. G. Tait, Maxwell implied that Tyndall had "martyred his scientific authority," adding: "If he writes it in a dry manner it is bad enough, but the harm is confined to students. But if he seasons it for the public and the public swallows it, then it is a sad misuse of words to say that this is useful work."<sup>6</sup> More playful, but perhaps equally pointed, was the "Tyndallic Ode" that Maxwell composed to parody Tyndall's very successful lecture style. Maxwell's Tyndallic lecturer began with some showy demonstration experiments, then resolved that "These transient facts / These fugitive impressions / Must be transformed by mental acts / To permanent possessions," and finally proceeded to construct a metaphysical tower of Babel upon his experimental sand:

Go to! prepare your mental bricks,  
Fetch them from every quarter,  
Firm on the sand your basement fix  
With best sensation mortar.  
The top shall rise to heaven on high—  
Or such an elevation,  
That the swift whirl with which we fly  
Shall conquer gravitation.<sup>7</sup>

Scientific naturalism sprang up in Britain in defense of Darwin's theory of evolution, but its prominent spokesmen included the physicist Tyndall and the mathematician W. K. Clifford as well as biologists like Huxley, and its rhetoric was saturated with concepts and terminology from mechanics. Naturalism was in part an instrument of professionalism, and biologists felt far less threatened by the scientific imperialism of physics than by the Anglican clergymen who insisted on bringing biology within the domain of natural theology and, worse, who occupied

<sup>5</sup> *Ibid.*, pp. 305-306.

<sup>6</sup> See Maxwell comment on galleys of Tait rebuttal to Tyndall in *JCMP* 7655 III d/5.

<sup>7</sup> Maxwell, pp. 635-636.

liberty which is not disorder,  
than the other, but more.<sup>5</sup>

conservative men like Maxwell  
ritings of the Victorian scientific  
tten along well with T. H. Hux-  
he found John Tyndall's ideas  
ommentary inspired by some re-  
l that Tyndall had "martyred his  
rites it in a dry manner it is bad  
idents. But if he seasons it for the  
it is a sad misuse of words to say  
ul, but perhaps equally pointed,  
ll composed to parody Tyndall's  
s Tyndallic lecturer began with  
ents, then resolved that "These  
ssions / Must be transformed by  
s," and finally proceeded to con-  
pon his experimental sand:

ental bricks,  
very quarter,  
basement fix  
a mortar.  
eaven on high—  
on,  
th which we fly  
itation.<sup>7</sup>

ritain in defense of Darwin's the-  
pokesmen included the physicist  
. Clifford as well as biologists like  
l with concepts and terminology  
part an instrument of profession-  
ned by the scientific imperialism  
men who insisted on bringing bi-  
eology and, worse, who occupied

ttal to Tyndall in JCMP 7655 III d/5.

scarce scientific posts when dedicated dissenting scientists had difficulty finding suitable positions. The appeal to physics embodied a drive to sever ties with natural theology, to remove biology from the empire of teleology. The object, as Huxley put it, was to "reduce all scientific problems, except those which are purely mathematical, to questions of molecular physics."<sup>8</sup> Huxley was appalled by the audacity of those who claimed authority to judge scientific theories such as Darwin's on the basis of what amounted to "sacerdotal pretensions,"<sup>9</sup> and Tyndall, displaying equal sensitivity to the purity of science and the sanctity of the scholar's turf, boldly announced the "impregnable position of science" as follows: "We claim, and we shall wrest from theology the entire domain of cosmological theory. All schemes and systems which thus infringe upon the domain of science must, in so far as they do this, submit to its control, and relinquish all thought of controlling it."<sup>10</sup>

If natural science was to implement these ambitious claims, it could afford to leave no gaps, and the idea of chance in science was viewed with skepticism by Huxley and Tyndall. One reason for opposition to Darwin's theory was that it replaced teleological purpose by chance variation—what John Herschel was reputed to have called "the law of higgledy-piggledy."<sup>11</sup> By "chance," Darwin of course meant undirected, not uncaused, but his defenders were nonetheless disturbed by the absence of a law of biological variation. Thus Huxley wrote to Hooker in 1861: "Because no law has yet been made out, Darwin is obliged to speak of variation as if it were spontaneous or a matter of chance, so that the bishops and superior clergy generally (the only real atheists and believers in chance left in the world) gird at him as if he were another Lucretius."<sup>12</sup>

This was by no means the last mention in late Victorian scientific discussion of the ancient atomists or of issues involving chance and mechanical determinism with which they were habitually associated. John

<sup>8</sup> T. H. Huxley, "The Scientific Aspects of Positivism," in *Lay Sermons, Addresses, and Reviews* (London, 1895), p. 144.

<sup>9</sup> Huxley, "On the Hypothesis that Animals are Automata, and its History" (1874), in *Collected Essays* (9 vols., New York, 1968), vol. 1, p. 249. See also Frank M. Turner, "The Victorian Conflict Between Science and Religion: A Professional Dimension," *Isis*, 69 (1978), 356-376.

<sup>10</sup> John Tyndall, "The Belfast Address" (1874), in George Bassala et al., eds., *Victorian Science* (New York, 1970), 436-478, pp. 474-475.

<sup>11</sup> David L. Hull, *Darwin and his Critics: The Reception of Darwin's Theory of Evolution by the Scientific Community* (Cambridge, Mass., 1973), pp. 7, 61.

<sup>12</sup> Leonard Huxley, *Life and Letters of Thomas H. Huxley* (2 vols., New York, 1901), vol. 1, p. 245.



Tyndall found it possible to enlist ancient atomism in the naturalist cause by sidestepping Lucretius, whose *De rerum natura* contained the troubling doctrine of the swerve, and invoking directly his predecessor Democritus, who had thoughtfully refrained from burdening the popular historian with any surviving works. In Tyndall's controversial 1874 address to the British Association in Belfast, he listed six fundamental scientific propositions of Democritus and pronounced the first five of them "a fair general statement of the atomic philosophy as now held." These were the following:

1. From nothing comes nothing. Nothing that exists can be destroyed. All changes are due to the combination and separation of molecules.
2. Nothing happens by chance. Every occurrence has its cause from which it follows by necessity.
3. The only existing things are the atoms and empty space, all else is mere opinion.
4. The atoms are infinite in number and infinitely various in form; they strike together, and the lateral motions and whirlings which thus arise are the beginnings of worlds.
5. The varieties of all things depend upon the varieties of their atoms, in number, size, and aggregation.<sup>13</sup>

Tyndall's line of thinking was more troubling to Maxwell even than Buckle's since it was precisely one of Maxwell's specialties, atomism, that was dressed up to support this deterministic and materialistic doctrine. Maxwell had long held that dynamical explanation represented a scientific ideal, and he had written just before the storm broke, in 1850, that "if we know what is at any assigned point of space at any assigned instant of time, we may be said to know all the events in Nature. We cannot conceive any other thing which it would be necessary to know."<sup>14</sup> It was not a little discomfiting to see these ideas applied vigorously to life and mind, in explicit opposition to religion. Maxwell preferred to cast his lot with Lucretius:

When Lucretius wishes us to form a mental representation of the motion of atoms, he tells us to look at a sunbeam shining through a darkened room (the same instrument of research by which Dr Tyndall makes visible to us the dust we breathe), and to observe the

<sup>13</sup> Tyndall, "Belfast Address" (n. 10), p. 443. See also Frank M. Turner, "Lucretius among the Victorians," *Victorian Studies*, 16 (1972-73), 329-348; and Turner, *The Greek Heritage in Victorian Britain* (New Haven, 1981).

<sup>14</sup> Maxwell, p. 238.

ancient atomism in the naturalists' use of *De rerum natura* contained the idea of invoking directly his predecessor and refrained from burdening the popular imagination. In Tyndall's controversial 1874 lecture at Belfast, he listed six fundamental principles and pronounced the first five of them the atomic philosophy as now held."

Nothing that exists can be decomposed by combination and separation of matter by chance. Every occurrence has a cause, not necessity. 3. The only existing space, all else is mere opinion. 4. Matter is not and infinitely various in form; all motions and whirlings which exist in the world. 5. The varieties of all things are determined by atoms, in number, size, and ag-

are troubling to Maxwell even than the limitations of Maxwell's specialties, atomism, deterministic and materialistic dynamical explanation represented a threat just before the storm broke, in 1856, at the signed point of space at any assigned time. We cannot know all the events in Nature. We must accept the fact which it would be necessary to admit in order to see these ideas applied vigorously in opposition to religion. Maxwell pre-

in a mental representation of the world as it is, at a sunbeam shining through the argument of research by which Dr. Johnson (the man we breathe), and to observe the

See also Frank M. Turner, "Lucretius among the ancients," pp. 329-348; and Turner, *The Greek Heritage in*

## Statistical Uncertainty

motes which chase each other in all directions through it. This motion of the visible motes, he tells us, is but a result of the far more complicated motion of the invisible atoms, which knock the motes about. In his dream of nature, as Tennyson tells us he

"Saw the flaring atom streams  
And torrents of her myriad universe  
Running along the illimitable inane,  
Fly on to clash together again, and make  
Another and another frame of things  
For ever."

And it is no wonder that he should have attempted to burst the bonds of Fate by making his atoms deviate from their courses at quite uncertain times and places, thus attributing to them a kind of irrational free will, which on his materialistic theory is the only explanation of that power of which we ourselves are conscious.<sup>15</sup>

Maxwell certainly did not wish to imply that natural science should be constrained to support certain religious doctrines. "The rate of change of scientific hypotheses is naturally much more rapid than that of Biblical interpretations," he told a clergyman in 1876, "so that if an interpretation is founded on such an hypothesis, it may help to keep the hypothesis above ground long after it ought to be buried and forgotten."<sup>16</sup> He thought it appropriate for Christians to seek to harmonize their science with their faith, but insisted that the results of this effort were valid only for the individual involved, and only for a limited time.<sup>17</sup> Arguments about the inconsistency of contemporary science with religion seemed to him to violate this dictum, proving, as his friends P. G. Tait and Balfour Stewart explained, that the opponents of faith, and not religious men, are the true dogmatists.<sup>18</sup> "No mind ever delighted more in speculation," observed Lewis Campbell of Maxwell, "and yet none was ever more jealous of the practical application or the popular dissemination of what appeared to him as crude and half-baked theories about the highest subjects."<sup>19</sup> Karl Pearson came away from a

<sup>15</sup> Maxwell, "Molecules," in *SP*, vol. 2, p. 373.

<sup>16</sup> Maxwell, p. 394.

<sup>17</sup> *Ibid.*, p. 405.

<sup>18</sup> Peter Guthrie Tait and Balfour Stewart, *The Unseen Universe, or Physical Speculations on a Future State* (London, 1875), p. v.

<sup>19</sup> Maxwell, p. 322.



Cambridge examination by Maxwell for the Smith's prize with a less charitable view:

The conversation turned on Darwinian evolution; I can't say how it came about, but I spoke disrespectfully of Noah's flood. Clerk Maxwell was instantly aroused to the highest pitch of anger, reproving me for want of faith in the Bible. I had no idea at the time that he had retained the rigid faith of his childhood, and was, if possible, a firmer believer than Gladstone in the accuracy of Genesis.<sup>20</sup>

It was in this context that Maxwell developed his ideas about the statistical character of the second law of thermodynamics, and more generally about the inescapable imperfection of human knowledge. Maxwell first pointed out that his dynamical theory of gases implied the possibility of violating the second law in 1867, in a playful letter to P. G. Tait. There he introduced the "very observant and neat-figured being," later dubbed by William Thomson Maxwell's "demon," whose mission was to "pick a hole" in the second law. The demon needed merely to be set to guard a small hole in the elastic wall between gases at different temperatures and it could—by allowing only the most energetic molecules to pass from the cold to the hot side, and only the least energetic in the opposite direction—cause heat to flow from a cold gas to a warm one. Notwithstanding Thomson's name for this fictional creature, which Maxwell found objectionable, no supernatural powers were required, but only an exaggerated level of ordinary ones.<sup>21</sup>

Maxwell's invention implied that some physical principles, among them the second law, were really as much attributes of human perceptions as of nature itself. He wrote a decade later in the *Encyclopaedia Britannica* under the heading "Diffusion":

The idea of dissipation of energy depends on the extent of our knowledge. Available energy is energy which we can direct into any desired channel. Dissipated energy is energy which we cannot lay hold of and direct at pleasure, such as the energy of the confused agitation of molecules which we call heat. Now, confusion, like

<sup>20</sup> Karl Pearson, "Old Tripos Days at Cambridge, as seen from another Viewpoint," *Mathematical Gazette*, 20 (1936), 27-36.

<sup>21</sup> The original letter is printed in C. G. Knott, *Life and Scientific Work of Peter Guthrie Tait* (Cambridge, 1911), pp. 213-214. See also the undated letter of Maxwell to Tait in *ibid.*, pp. 214-215.

or the Smith's prize with a less

an evolution; I can't say how  
tfully of Noah's flood. Clerk  
e highest pitch of anger, re-  
ible. I had no idea at the time  
his childhood, and was, if pos-  
one in the accuracy of Gene-

eveloped his ideas about the sta-  
hermodynamics, and more gen-  
ion of human knowledge. Max-  
cal theory of gases implied the  
in 1867, in a playful letter to P.  
very observant and neat-figured  
son Maxwell's "demon," whose  
econd law. The demon needed  
in the elastic wall between gases  
—by allowing only the most en-  
to the hot side, and only the least  
use heat to flow from a cold gas  
son's name for this fictional crea-  
le, no supernatural powers were  
of ordinary ones.<sup>21</sup>  
ome physical principles, among  
uch attributes of human percep-  
cade later in the *Encyclopaedia*  
on":

depends on the extent of our  
y which we can direct into any  
s energy which we cannot lay  
as the energy of the confused  
ll heat. Now, confusion, like

, as seen from another Viewpoint," *Math-*

*Life and Scientific Work of Peter Guthrie*  
undated letter of Maxwell to Tait in *ibid.*,

## Statistical Uncertainty

the correlative term order, is not a property of material things in themselves, but only in relation to the mind that perceives them. A memorandum-book does not, provided it is neatly written, appear confused to an illiterate person, or the owner who understands it thoroughly, but to any other person able to read it appears to be inextricably confused. Similarly the notion of dissipated energy could not occur to a being who could not turn any of the energies of nature to his own account, or to one who could trace the motion of every molecule and seize it at the right moment. It is only to a being in the intermediate stage, who can lay hold of some forms of energy while others elude his grasp, that energy appears to be passing from the available to the dissipated state.<sup>22</sup>

Maxwell first adumbrated a connection between the indeterminacy of certain thermodynamic principles and their statistical character in 1868, when he compared the tendency for gas molecules to assume the normal velocity distribution to the mixing of black and white balls in a box. Two years later he wrote to the young physicist Rayleigh that the second law had "the same degree of truth as the statement that if you throw a tumblerful of water into the sea you cannot get the same tumblerful of water out again."<sup>23</sup> Maxwell began to develop an argument about the wider implications of the statistical method in physics in the inaugural lecture he delivered in 1871 upon becoming head of the new Cavendish Laboratory at Cambridge. There he remarked, explicitly and publicly, that predictions based on statistical knowledge were inherently uncertain. The gas laws evidently were of a different character from dynamical principles, yielding a form of knowledge whose implications he thought had wide interest. Maxwell pointed immediately to the vexed question of human freedom, arguing:

. . . the statistical method . . . , which in the present state of our knowledge is the only available method of studying the properties of real bodies, involves an abandonment of strict dynamical principles, and an adoption of the mathematical methods belonging to

<sup>22</sup> Maxwell, "Diffusion," in *SP*, vol. 2, p. 646.

<sup>23</sup> R. J. Strutt, Fourth Baron Rayleigh, *Life of John William Strutt, Third Baron Rayleigh* (Madison, Wisc., augmented ed., 1968), p. 47. See also Stephen Brush, "Randomness and Irreversibility," in *The Kind of Motion We Call Heat* (2 vols., Amsterdam, 1976), 543-654, p. 590, *passim*; also Brush, "Irreversibility and Indeterminism," chap. 2 of his *Statistical Physics and the Atomic Theory of Matter from Boyle and Newton to Landau and Onsager* (Princeton, 1981).



the theory of probability. It is probable that important results will be obtained by the application of this method, which is as yet little known and is not familiar to our minds. If the actual history of Science had been different, and if the scientific doctrines most familiar to us had been those which must be expressed in this way, it is possible that we might have considered the existence of a certain kind of contingency a self-evident truth, and treated the doctrine of philosophical necessity as a mere sophism.<sup>24</sup>

By 1873 Maxwell had developed this line of reasoning into a full argument against the increasingly threatening doctrine of mechanical determinism. His aim was not to use physics to demonstrate the existence of human freedom, for he adhered to the Common Sense tenet that belief in free will arises naturally from reflection on the mind's own activity. He carried out his examination of physical knowledge precisely in order to show that known scientific principles in fact proved nothing on this vital issue. Maxwell did not, however, deny natural science all relevance to metaphysical issues. He suggested that by showing what freedom could not be, physics offered valuable guidance in identifying what it was. The inherent interest of this project, as well as the need to demonstrate the possibility of free action that did not violate any laws of physics, inspired Maxwell to devote considerable attention to the physical interpretation of free will.

Already in 1862 Maxwell had observed to Lewis Campbell that the conservation of energy permitted the soul to be switchman, but not mover of the body. "There is action and reaction between body and soul," he wrote, "but it is not of a kind in which energy passes from one to the other." The direction which the soul could give to the energy of the body was comparable, he thought, to the relation between trigger and gun, or pointsman and train. Maxwell disclaimed any pretense to having solved this issue—"It is well that it will go, and that we remain in possession, though we do not understand it,"<sup>25</sup>—but he was not long willing to rest secure in ignorance. The frequent invocation of conservation laws against human freedom convinced the defenders of metaphysical freedom of their obligation to demonstrate the possibility that the will could operate without any expenditure of energy.<sup>26</sup> The most

<sup>24</sup> Maxwell, "Introductory Lecture on Experimental Physics," in *SP*, vol. 2, p. 253.

<sup>25</sup> Maxwell, p. 336.

<sup>26</sup> As C. S. Peirce noted of Simon Newcomb, defenders of free will were strangely indifferent to violations of Newton's third law of motion. See Charles Sanders Peirce, "Variety and

ple that important results will  
s method, which is as yet little  
ds. If the actual history of Sci-  
cientific doctrines most famil-  
be expressed in this way, it is  
ered the existence of a certain  
uth, and treated the doctrine of  
phism.<sup>24</sup>

s line of reasoning into a full ar-  
ening doctrine of mechanical de-  
sics to demonstrate the existence  
he Common Sense tenet that be-  
flection on the mind's own activ-  
physical knowledge precisely in  
nciples in fact proved nothing on  
ver, deny natural science all rel-  
gested that by showing what free-  
able guidance in identifying what  
object, as well as the need to dem-  
it did not violate any laws of phys-  
derable attention to the physical

rved to Lewis Campbell that the  
soul to be switchman, but not  
and reaction between body and  
in which energy passes from one  
2 soul could give to the energy of  
t, to the relation between trigger  
xwell disclaimed any pretense to  
at it will go, and that we remain  
stand it,"<sup>25</sup>—but he was not long  
e frequent invocation of conser-  
onvinced the defenders of meta-  
o demonstrate the possibility that  
penditure of energy.<sup>26</sup> The most-

ental Physics," in *SP*, vol. 2, p. 253.

efenders of free will were strangely indiffer-  
See Charles Sanders Peirce, "Variety and

cited physicist's defense of free will was published anonymously in the *North British Review* in 1868 by Fleeming Jenkin, who evidently sent Maxwell the proofs in advance.<sup>27</sup> His argument, cast as a defense of the Lucretian doctrine of the swerve, was virtually the same as Maxwell's; Jenkin wrote that "if mind or will deflects matter as it moves, it may produce all the consequences claimed by the Wilful school, and yet it will add neither energy nor matter to the universe."<sup>28</sup>

The main incentive for Maxwell's reflections on the limits of statistical knowledge was his desire to show that freedom was not inconsistent with the laws of nature properly known by contemporary science. His fundamental argument was that the statistical method, the only means by which humans can attain general knowledge of a molecular universe, yields only generalizations about the mass of molecules and provides no information about individuals. He wrote in his 1871 textbook on heat:

It is therefore possible that we may arrive at results which, though they fairly represent the facts as long as we are supposed to deal with a gas in mass, would cease to be applicable if our faculties and instruments were so sharpened that we could detect and lay hold of each molecule and trace it through all its course.

For the same reason, a theory of the effects of education deduced from a study of the returns of registrars, in which no names of individuals are given, might be found not to be applicable to the experience of a schoolmaster who is able to trace the progress of each individual pupil.<sup>29</sup>

Hence the need to resort to statistics guaranteed not only that nothing was known of the particular circumstances of individual molecules but that even the laws of their motion might bear no determinate relation to the observable regularities of the mass.

Maxwell developed these arguments most fully in a paper that he read early in 1873 for Eranus, a club of past Cambridge Apostles, titled "Does the Progress of Physical Science tend to give any advantage to the

Uniformity" (1903), in Charles Hartshorne et al., eds., *Collected Papers of Charles Sanders Peirce* (8 vols., Cambridge, Mass., 1931-1958), vol. 6, 67-85, p. 70.

<sup>27</sup> See Jenkin's letter to Maxwell, 10 July 1868, in *JCMP*, 7655/II, Box 1.

<sup>28</sup> [Fleeming Jenkin], "The Atomic Theory of Lucretius," *North British Review*, 48 (1868), 111-128, p. 118. This paper was widely circulated, and quoted by, among others, William Thomson, "The Structure of Matter and the Unity of Science," in Bassala, *Victorian Science* (n. 10), 101-128, p. 109, and Tait and Stewart, *Unseen Universe* (n. 18), pp. 181-182.

<sup>29</sup> Maxwell, *Theory of Heat* (1871; London, 1904), pp. 315-316.



opinion of Necessity (or Determinism) over that of the Contingency of Events and the Freedom of the Will?" Here atomism, thermodynamics, and statistics, a trio of sources for those deterministic arguments seen as most compelling in mid-Victorian Britain, were turned on their heads and reinterpreted as evidence for the possibility of human freedom. Maxwell began by noting that the statistical method, which "has Laplace for its most scientific and Buckle for its most popular expounder," was by its nature an imperfect one, applicable precisely when the course of individual events cannot be charted or explained. Its use, he observed depends on an assumption "that the effects of widespread causes, though very different in each individual, will produce an average result on the whole nation, from a study of which we may estimate the character and propensities of an imaginary being called the Mean Man." Far from being unique to social science, however, this form of reasoning constituted the basis for "all our knowledge of matter," assuming that the molecular hypothesis was true. "A constituent molecule of a body has properties very different from those of the body to which it belongs," he continued, and "those uniformities which we observe in our experiments with quantities of matter containing millions of millions of molecules are uniformities of the same kind as those explained by Laplace and wondered at by Buckle arising from the slumping together of multitudes of causes each of which is by no means uniform with the others."<sup>30</sup>

This, evidently, left open a certain space of ignorance, within which the will could operate without its direct effects being perceived. In this respect "our free will is at best like that of Lucretius's atoms," extending only over an infinitesimal range. Under certain circumstances, however, these limitations might be transcended. Maxwell wrote: "It has been well pointed out by Professor Balfour Stewart that physical stability is the characteristic of those systems from the contemplation of which determinists draw their arguments and physical instability that of those living bodies, and moral instability that of those developable souls, which furnish to consciousness the conviction of free will." The mind, he indicated, was a system for generating and regulating instabilities, through which human freedom and moral responsibility are expressed. "In the course of this our mortal life, we more or less frequently find ourselves on a physical or moral watershed, where an imperceptible de-

<sup>30</sup> Maxwell, 434-444, pp. 438-439.

viation is sufficient to determine into which of two valleys we shall descend. The doctrine of free will asserts that in some cases the Ego alone is the determining cause."<sup>31</sup> The statistical character of knowledge insured that no finite observer could be in a position to refute this possibility.

A few years later, Maxwell encountered a refinement to this solution to the free-will problem that seemed to him more satisfactory. His source was a group of French and Belgian Catholic scientists who were similarly repelled by the deterministic materialism of Tyndall, and also of Comte's disciple Emile Littré and the American John Draper. As Mary Jo Nye has shown, Ignace Carbonelle, Joseph Delsaulx, and Julien Thirion had, like Maxwell, found in the kinetic gas theory justification for the belief that some measure of uncertainty and unpredictability characterized scientific laws. They were especially interested in the problem of Brownian motion, the irregular movements of tiny but visible particles suspended in fluids, which was first linked to the kinetic theory by Delsaulx. Maxwell found his inspiration in the work on fluid mechanics of another member of this group, Joseph Boussinesq, who showed in 1878 that under certain conditions the differential equations regulating a mechanical system should have multiple solutions at certain points of singularity, and hence that determinate forces might produce no uniquely determined motion. In that event, it would be possible for a "directing principle" such as the will to determine which possible solution actually occurred.<sup>32</sup>

Oddly enough, Maxwell developed this line of thinking most fully in a letter to Francis Galton, with whom he did not regularly correspond and whom he once characterized as a man "whose mission it seems to be to ride other men's hobbies to death."<sup>33</sup> Maxwell's comments, which appeared as a postscript to an otherwise brief and routine letter renewing his subscription to the Philosophical Club, were as follows:

Do you have any interest in Fixt Fate, Free Will, &c. If so Boussinesq [of hydrodynamic reputation] "Conciliation du veritable determinisme mécanique avec l'existence de la vie et de la liberté mo-

<sup>31</sup> *Ibid.*, pp. 440-441.

<sup>32</sup> See Mary Jo Nye, "The Moral Freedom of Man and the Determinism of Nature: The Catholic Synthesis of Science and History in the *Revue des questions scientifiques*," *BJHS*, 9 (1976), 274-292, pp. 277-281. The idea of a directing principle derives from A. A. Cournot and was revived by Boussinesq's mentor, St. Venant. Nye also mentions Claude Bernard in this connection.

<sup>33</sup> Maxwell, p. 390.



rale" (Paris, 1878) does the whole business by the theory of the singular solutions of the differential equations of motion. Two other Frenchmen have been working on the same or similar tracks. Cournot (now dead) and de St. Venant [of elastic reputation *Torsion of Prism &c.*].

Another, also in the engineering line of research. Philippe Breton seems to me to be somewhat like minded with these.

There are certain cases in which a material system, when it comes to a phase in which the particular path which it is describing coincides with the envelope of all such paths may either continue in the particular path or take to the envelope (which in these cases is also a possible path) and which course it takes is not determined by the forces of the system (which are the same for both cases) but when the bifurcation of path occurs, the system, ipso facto, invokes some determining principle which is extra physical (but not extra natural) to determine which of the two paths it is to follow.

When it is on the enveloping path it may at any instant, at its own sweet will, without exerting any force or spending any energy, go off along that one of the particular paths which happens to coincide with the actual condition of the system at that instant.

In most of the former methods Dr. Balfour Stewart's, &c. there was a certain small but finite amount of travail decrochant or trigger-work for the will to do. Boussinesq has managed to reduce this to mathematical zero, but at the expense of having to restrict certain of the arbitrary constants of the motion to mathematically definite values, and this I think will be found in the long run very expensive. But I think Boussinesq's method is a very powerful one against metaphysical arguments about cause and effect and much better than the insinuation that there is something loose about the laws of nature, not of sensible magnitude but enough to bring her round in time.<sup>34</sup>

That Maxwell's argument about the imperfection of physical knowledge and even the possible incompleteness of mechanical laws should have developed out of his thoughts on the implications of statistics for physics seems completely plausible—perhaps too plausible—in retrospect. To be sure, Maxwell himself came to see this connection as a natural one. He often noted how much looseness there was in statistical

<sup>34</sup> Maxwell to Galton, 26 Feb. 1879, FGP, folder 191. I have silently closed a parenthesis in the first paragraph. The material inside brackets is Maxwell's.

business by the theory of the  
equations of motion. Two  
on the same or similar tracks.  
ant [of elastic reputation Tor-

ine of research. Philippe Bre-  
minded with these.

a material system, when it  
lar path which it is describing  
ch paths may either continue  
velope (which in these cases  
urse it takes is not determined  
e the same for both cases) but  
the system, ipso facto, invokes  
s extra physical (but not extra  
vo paths it is to follow.

h it may at any instant, at its  
force or spending any energy,  
r paths which happens to co-  
e system at that instant.

. Balfour Stewart's, &c. there  
t of travail decrochant or trig-  
sq has managed to reduce this  
ense of having to restrict cer-  
nition to mathematically def-  
ound in the long run very ex-  
ethod is a very powerful one  
at cause and effect and much  
is something loose about the  
tude but enough to bring her

imperfection of physical knowl-  
ness of mechanical laws should  
the implications of statistics for  
erhaps too plausible—in retro-  
e to see this connection as a nat-  
ooseness there was in statistical

r 191. I have silently closed a parenthesis  
s Maxwell's.

## Statistical Uncertainty

generalizations, if not in the laws of nature themselves. Using one of Galton's favorite similes, he remarked in an unpublished manuscript that "the population of a watering-place, considered as a mere number, varies in the same way whether its visitors return to it season after season or whether the annual flock consists each year of fresh individuals."<sup>35</sup> He was right, of course, but it should not be forgotten that these ideas were new, that the imperfection of statistical knowledge had received scarcely any attention before 1857 except by those who rejected it entirely, and that Maxwell began pursuing this line of thought only when he was provoked by the vehement expression of statistical and mechanical determinism by Buckle and Tyndall.

The persuasive force of Maxwell's conclusions on statistical knowledge is evident from the circumstance that Boltzmann, who was completely unsympathetic to probabilism in thermodynamics, was compelled by problems arising within statistical gas theory to move a long way in the same direction. Still, statistics did not require the embrace of non-mechanical causation, much less of indeterminism, for it was always possible to believe the underlying phenomena to be mechanically determined, and only the large-scale numerical regularities probabilistic. Francis Galton saw no reason to move away from determinism on account of the statistical character of knowledge, as is in some way indicated by his unresponsive response to Maxwell's unsolicited missive, where he noted that in his own introspective psychological experiments he had been "almost frightened to find how distinctly cause and effect seem to govern everything."<sup>36</sup> Boltzmann eventually accepted the im-

<sup>35</sup> Undated manuscript, JCMP, 7655/V ff11.

<sup>36</sup> Galton to Maxwell, 27 Feb. 1879 in JCMP, 7655/II (Box I). The relevant text of Galton's reply is as follows:

Very many thanks for the free will &c.—After all, does the question not coincide in final principle with that of unstable equilibrium?—I am rather busy just now with experiments on the workings of my own mind, and am almost frightened to find how distinctly cause & effect seem to govern everything.—If you happen to see the forthcoming "Nineteenth Century" (March) & care to look at a short paper in it by me "Psychometric facts"—it will as you will see describe something of what I mean, & how I continue to drag ideas that are almost out of the reach of consciousness into full light.

I look forward with infinite interest to—moreover—to hear Sir W. Thomson's exposition of your sorting demon at the Royal Institution. I have little doubt that some will go to see art illustrations of the medieval Devil.

In "Free Will—Observations and Inferences," *Mind*, 9 (1884), 406-413, p. 412, Galton concluded even more decisively that "man is little more than a conscious machine," and that "we must understand the word 'spontaneity' in the same sense that a scientific man understands the word 'chance.' He thereby affirms his ignorance of the precise causes of an event, but he does not in any way deny the possibility of determining them."



### Statistical Uncertainty

plications of his statistical approach, but always emphasized the positive content and power of its accomplishments, not its limitations.

#### BOLTZMANN, STATISTICS, AND IRREVERSIBILITY

The kinetic gas theory constituted Boltzmann's life work, and the language and concepts of probability theory were central to his research in this field from the beginning. He was never comfortable, however, with the idea that the form of his mathematics implied any indeterminacy in the resulting laws. Boltzmann would have derived no satisfaction from "picking a hole" in the second law of thermodynamics, as Maxwell had. He always stressed instead the certainty of science:

A precondition of all scientific knowledge is the principle of the complete (*eindeutig*) determination of all natural processes, or, as applied to mechanics, the complete determination of all movements. This principle declares, that the movements of a body do not occur purely accidentally, going sometimes here, sometimes there, but that they are completely determined by the circumstances to which the body is subject.<sup>37</sup>

As we have seen, Boltzmann invoked the analogy of gas theory with social statistics in order to bolster, not undermine, the certainty of his scientific conclusions. Nevertheless, he did not glide over difficulties—at least not for long—but faced them with resilience and creativity. His faith in molecular models was so deep as to be virtually a matter of principle—he thought continuity was meaningful even in mathematics only as the limit of finite differentials<sup>38</sup>—and he held to atomism even when he found himself compelled by it to concede that certain macroscopic laws, such as the second law of thermodynamics, were only statements of high probability.

Boltzmann was an Austrian, the son of a Viennese tax official. He was born in 1844, about a decade later than most of the holistic historical

<sup>37</sup> Boltzmann, "Über die Grundprinzipien und Grundgleichungen der Mechanik" (1899), in *PS*, pp. 276-277.

<sup>38</sup> It may be noted in this connection that, according to his friend and Vienna colleague Franz Clemens Brentano, Boltzmann thought Joseph Bertrand's paradox of probability (see n. 44, chap. 3, above) would be cleared up once the fallacy of assuming a continuum was recognized. See Brentano, "Von der Unmöglichkeit absoluten Zufalls" (1916) in *Versuch über die Erkenntnis* (Hamburg, 1970), p. 141.

USD 57.80

THE RISE OF  
STATISTICAL THINKING

1820-1900

Theodore M. Porter

LIBRARY OF THE  
CENTRAL EUROPEAN

1986

PRINCETON UNIVERSITY PRESS