**M1. BASIC ALGEBRA 1**

**Course coordinator:** Pal Hegedus

**No. of Credits:** 3, and no. of ECTS credits: 6

**Time Period of the course**: Fall Semester

**Prerequisites:**linear algebra, introductory abstract algebra

**Course Level:** introductory MS

**Brief introduction to the course:**

Basic concepts and theorems are presented. Emphasis is put on familiarizing with the aims and methods of abstract algebra. Interconnectedness is underlined throughout. Applications are presented.

**The goals of the course:**

One of the main goals of the course is to introduce students to the most important concepts and fundamental results in abstract algebra. A second goal is to let them move confidently between abstract and concrete phenomena.

**The learning outcomes of the course:**

By the end of the course, students areexperts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

**More detailed display of contents (week-by-week):**

1. Groups: permutations groups, orbit-stabilizer theorem, cycle notation, conjugation, conjugacy classes of S\_n, odd/even permutations,
2. commutator subgroup, free groups, geerators and relations, Dyck’s theorem,
3. solvable and simple groups, simplicity of A\_n, classical linear groups,
4. Polynomials: Euclidean Algorithm, uniqueness of factorisation, Gauss Lemma, cyclotomic polynomials,
5. polynomials in several variables, homogeneous polynomials, symmetric polynomials, formal power series, Newton's Formulas,
6. Sturm’s Theorem on the number of real roots of a polynomial with real coefficients.
7. Rings and modules: simplicity of matrix rings, quaternions, Frobenius Theorem, Wedderburn’s Theorem,
8. submodules, homomorphisms, direct sums of modules, free modules,
9. chain conditions, composition series.
10. Partially ordered sets and lattices: Hasse-diagram, chain conditions, Zorn Lemma, lattices as posets and as algebraic structures,
11. modular and distributive lattices, modularity of the lattice of normal subgroups, Boolean algebras, Stone Representation Theorem.
12. Universal algebra: subalgebras, homomorphisms, direct products, varieties, Birkhoff Theorem.

Optional topics:

Resultants, polynomials in non-commuting variables, twisted polynomials, subdirect products, subdirectly irreducible algebras, subdirect representation. Categorical approach: products, coproducts, pullback, pushout, functor categories, natural transformations, Yoneda lemma, adjointfunctors.

**References**:

1. P J Cameron, Introduction to Algebra, Oxford University Press, Oxford, 2008.

2. N Jacobson, Basic Algebra I-II, WH Freeman and Co., San Francisco, 1974/1980.

3. I M Isaacs, Algebra, a graduate course, Brooks/Cole Publishing Company, Pacific Grove, 1994